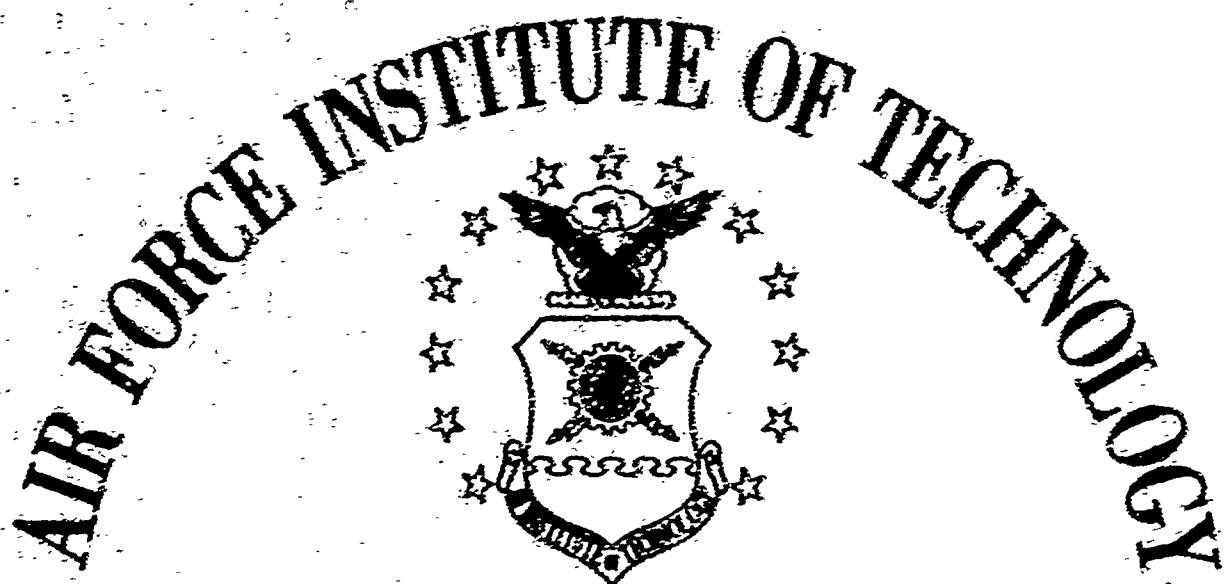


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UNITED STATES AIR FORCE

A TRANSPORT CALCULATION OF THE FLUX  
IN THE NUCLEAR ENGINEERING  
TEST REACTOR TEST-CELLS

Lt. Lee H. Livingston

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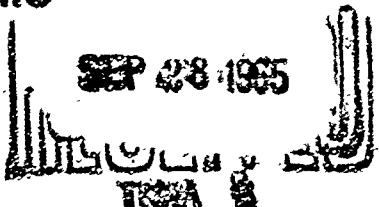
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WRIGHT-PATTERSON AIR FORCE BASE, OHIO

SEP 28 1965



A TRANSPORT CALCULATION OF THE FLUX  
IN THE NUCLEAR ENGINEERING  
TEST REACTOR TEST-CELLS

THESIS

Presented to the Faculty of the School of Engineering of  
the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by

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Lieutenant USAF  
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June 1965

Preface

The Nuclear Engineering Test Reactor at Wright-Patterson Air Force Base is somewhat unique in that it was designed primarily to test the effects of high-intensity radiation on materials. Two large voids (test-cells) on the east and west faces of the core provide the means of conducting such experiments on very large items. The calculation of the flux in these test-cells is hence of the utmost importance. I decided to write my own code for this calculation, although other discrete  $S_n$  codes are available, since the principal purpose of a master's thesis is that of a learning device.

When this project was first begun, I had planned to calculate the flux in the test-cells of the NETR in three dimensions. After the program had been debugged and some preliminary runs made in two dimensions, it was found that the running time which would be involved was too expensive for a project of this type. Hence the remainder of my thesis period was spent in trying to refine the program sufficiently to get reasonable results in xy-geometry.

I wish to thank Drs. Carlson, Lathrop, and Lee of Los Alamos Scientific Laboratory for their kindness in answering my letters concerning difficulties I encountered in the theory; Dr. Bridgman of AFIT for his assistance and criticisms; and Lt. Max Thompson of AFIT who executed the PDQ code for me. Finally, I wish to express my spe-

cial thanks to my wife Marti who typed this thesis.

Lee H. Livingston

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Abstract

This report describes a calculation of the neutron flux in the test-cells of the Nuclear Engineering Test Reactor at Wright-Patterson Air Force Base, Ohio, using the discrete  $S_n$  approximation to the transport equation in xy-geometry. The calculation was done by writing a computer program, S4C40, for the IBM 7094 using three energy groups and 1600 mesh points. Fast cross-sections were generated with the General Atomics code GAM-I, and thermal cross-sections were calculated by hand assuming a Maxwell-Boltzmann distribution. The results, which were compared with the multigroup, two-dimensional diffusion theory code PDQ, show a significantly higher thermal flux over the entire reactor and a more slowly decreasing flux for all groups in the test-cell.

A TRANSPORT CALCULATION OF THE FLUX  
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TEST REACTOR TEST-CELLS

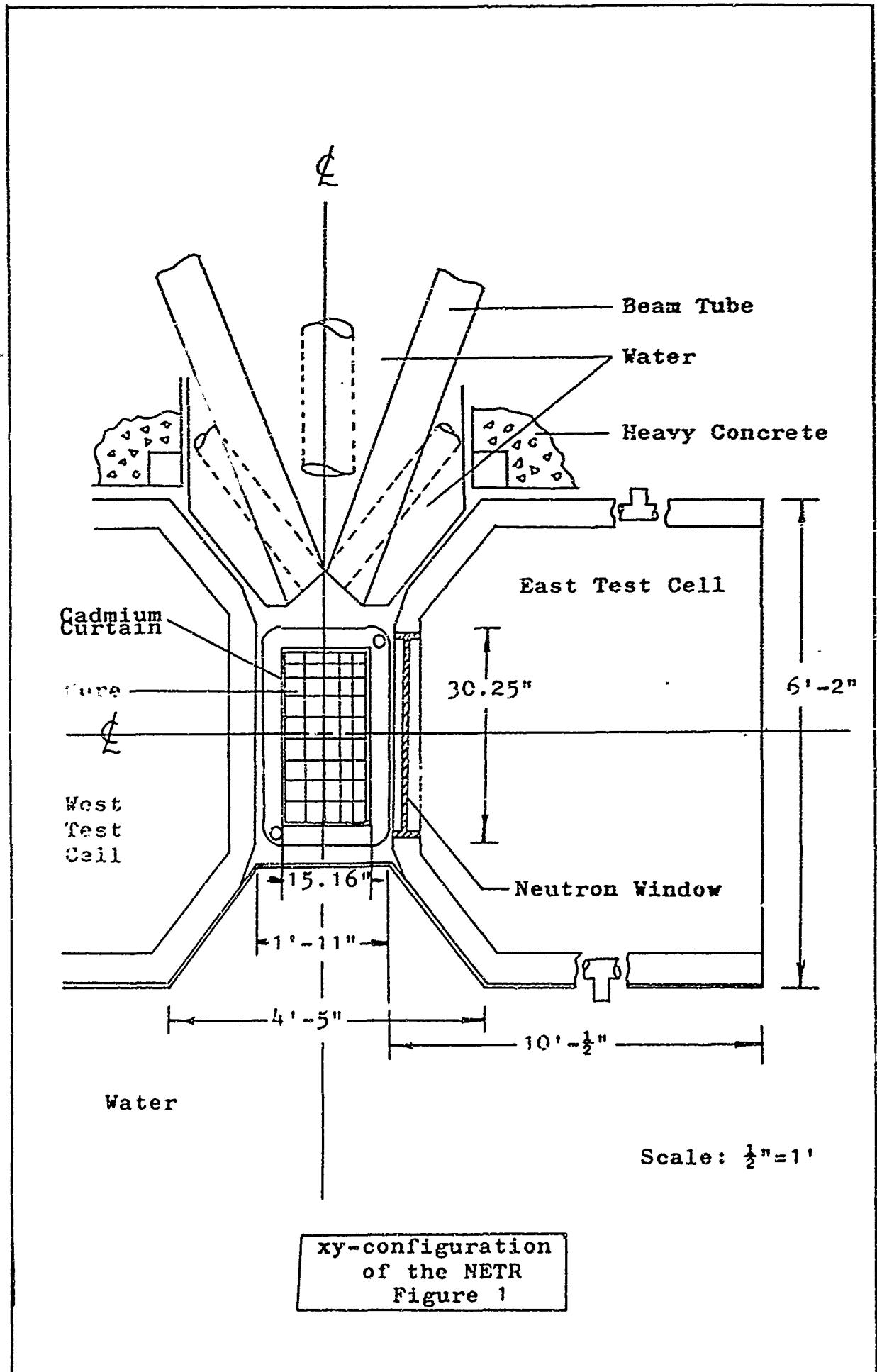
I. Introduction

This paper describes a calculation of the neutron flux in the test-cells of the Nuclear Engineering Test Reactor at Wright-Patterson Air Force Base, Ohio, by means of the  $S_n$  approximation to the transport equation. The calculation was done using three energy groups and 1600 mesh points in a code written in Fortran for the IBM 7094 computer.

Statement of Problem

The NETR is a light-water moderated, thermal reactor with MTR-type fuel elements. On the east and west sides of the core are two large voids with lead and steel walls, surrounded by heavy concrete as shown in Figure 1. These cells are approximately ten feet deep, seven feet high, and seven feet wide.

The purpose of the test-cells is to provide a high-energy neutron flux environment for irradiation effects studies of various materials. The cells may be filled with various gases and are equipped with small gauge tracks to allow massive experimental apparatus to be inserted. A cadmium curtain is in juxtaposition with the



core to reduce the thermal flux from the spectrum entering the cells. In the region of the cadmium, which is a strongly absorbing medium, and in the test-cells, where one is everywhere less than one mean free path from a boundary, diffusion theory is inapplicable; hence, transport methods should be used to calculate the neutron flux.

#### Method of Attack

The transport equation is, in principle, capable of predicting the neutron flux as a function of position  $\vec{r}$ , energy  $E$ , and direction of travel  $\hat{\Omega}$ . Unfortunately, only a few special cases can be solved in closed form (Ref 6), and for most engineering problems one must turn to some means of approximating the solutions.

Two of the more common methods are the  $P_n$  (Ref 14:336) and  $B_n$  (Ref 9) approximations. These methods are versatile and accurate but involve a great deal of mathematical complexity. The discrete  $S_n$  approximation gives good results (Ref 5:539) and is a great deal simpler in form. It can be programmed easily for a computer. The discrete  $S_n$  difference equations are developed in Chapter II. Chapter III describes the calculation of the constants in the problem and Chapter IV the results obtained. The conclusions drawn from this work and recommendations for future work are given in Chapter V.

II. Derivation of the Discrete  $S_n$  Approximation  
in XY-Geometry

The derivation of the discrete  $S_n$  approximation to the transport equation begins with the steady-state transport equation for monoenergetic neutrons (Refs 4, 16:219) of energy group  $g$ , which is

$$\hat{\Omega} \cdot \vec{\nabla} N^g + \sum_T^0 N^g = S^g \quad (1)$$

where  $N = N(x, y, \mu, \phi)$  is the neutron flux and  $S = S(x, y)$  is the source term. The transport equation is a statement of the conservation of neutrons. One equates losses due to spatial leakage  $(\hat{\Omega} \cdot \vec{\nabla} N^g)$  and absorption and scattering collisions  $(\sum_T^0 N^g)$ , to the source from fission, external sources, and scatter into the group and direction under consideration. The expansion of the dot product of the direction vector  $\hat{\Omega}$  with the gradient operator  $\vec{\nabla}$  in rectangular geometry gives

$$\left[ \sin \theta \cos \phi \frac{\partial}{\partial X} + \sin \theta \sin \phi \frac{\partial}{\partial Y} + \sum_T^0 \right] N = S \quad (2)$$

If one defines  $\cos \theta$  as  $\mu$ , then  $\sin \theta = \sqrt{1 - \mu^2}$

One now operates on the transport equation with the operator

$$\frac{1}{\Delta X \Delta Y \Delta \mu \Delta \phi} \int_{X_{j-1}}^{X_j} \int_{Y_{j-1}}^{Y_j} \int_{\mu_{n-1}}^{\mu_n} \int_{\phi_{l-1}}^{\phi_l} \langle \rangle \quad (3)$$

to create a difference equation in the flux as a function

of the four variables  $x$ ,  $y$ ,  $\mu$ , and  $\phi$ . Consider the first term of the resulting equation

$$\frac{1}{\Delta x \Delta y \Delta \mu \Delta \phi} \int_{x_{i-1}}^{x_i} dx \int_{y_{j-1}}^{y_j} dy \int_{\mu_{n-1}}^{\mu_n} d\mu \int_{\phi_{l-1}}^{\phi_l} d\phi \left[ \sqrt{1-\mu^2} \cos \phi \frac{\partial}{\partial x} N(x, y, \mu, \phi) \right] \quad (4)$$

$$= \frac{1}{\Delta x \Delta y \Delta \mu \Delta \phi} \int_{y_{j-1}}^{y_j} dy \int_{\mu_{n-1}}^{\mu_n} d\mu \int_{\phi_{l-1}}^{\phi_l} d\phi \left\{ \sqrt{1-\mu^2} \cos \phi \left[ N(x_i, y, \mu, \phi) - N(x_{i-1}, y, \mu, \phi) \right] \right\} \quad (5)$$

where the assumption has been made that the integral of the partial derivative of a function equals the change in the function over the interval of integration. The integration over the other three variables assumes that the integral of a function equals the function evaluated at an average value of the variable times the length of the interval. Hence one may write.

$$\frac{\Delta \phi_l}{\Delta x \Delta y \Delta \mu \Delta \phi} \int_{y_{j-1}}^{y_j} dy \int_{\mu_{n-1}}^{\mu_n} d\mu \left\{ \sqrt{1-\mu^2} \langle \cos \phi_l \rangle \left[ N(x_i, y, \mu, \bar{\phi}_l) - N(x_{i-1}, y, \mu, \bar{\phi}_l) \right] \right\} \quad (6)$$

$$= \frac{\langle \cos \phi_\ell \rangle \Delta \mu}{\Delta x \Delta y \Delta \mu} \int_{Y_{j-1}}^{Y_j} dY \left\{ \langle \sqrt{1 - \mu_n^2} \rangle [N(X_i, Y, \bar{\mu}_n, \bar{\phi}_\ell) - N(X_{i-1}, Y, \bar{\mu}_n, \bar{\phi}_\ell)] \right\} \quad (7)$$

where the quantity in  $\langle \quad \rangle$  indicates an average value of that quantity taken over the range of integration. The resultant term is

$$\frac{\langle \cos \phi_\ell \rangle \langle \sqrt{1 - \mu^2} \rangle}{\Delta x \Delta y} [N(X_i, \bar{Y}_j, \bar{\mu}_n, \bar{\phi}_\ell) - N(X_{i-1}, \bar{Y}_j, \bar{\mu}_n, \bar{\phi}_\ell)] \Delta y \quad (8)$$

One now makes the approximation that

$$\langle \cos \phi_\ell \rangle \approx \cos \bar{\phi}_\ell \quad (9)$$

and

$$\langle \sqrt{1 - \mu_n^2} \rangle \approx \sqrt{1 - \bar{\mu}_n^2} \approx \eta_n \quad (10)$$

for some values of  $\bar{\mu}_n$  and  $\bar{\phi}_\ell$ . These values are determined later in the chapter. Thus Eq (8) can be written as

$$\frac{\eta_n \cos \bar{\phi}_\ell}{\Delta x} [N(X_i, \bar{Y}_j, \bar{\mu}_n, \bar{\phi}_\ell) - N(X_{i-1}, \bar{Y}_j, \bar{\mu}_n, \bar{\phi}_\ell)] \quad (11)$$

For notational simplification, write Eq (11) as

$\eta_n \cos \bar{\phi}_\ell [N_i - N_{i-1}] / \Delta x$  where the absence of a sub-

script implies the average value of that variable. By a similar procedure, the second term is found to be

$\eta_n \sin \bar{\phi}_\ell [N_j - N_{j-1}] / \Delta y$ . Operating on the third term, one

obtains

$$\frac{1}{\Delta X \Delta Y \Delta \mu \Delta \phi} \int_{X_{i-1}}^{X_i} \int_{Y_{j-1}}^{Y_j} \int_{\mu_{n-1}}^{\mu_n} \int_{\phi_{\ell-1}}^{\phi_\ell} \left[ \sum_T (X, Y) N(X, Y, \mu, \phi) \right] dX dY d\mu d\phi \\ = \sum_T (\bar{X}_i, \bar{Y}_j) N(\bar{X}_i, \bar{Y}_j, \bar{\mu}_n, \bar{\phi}_\ell) \equiv \sum_T \bar{N}_i \quad (12)$$

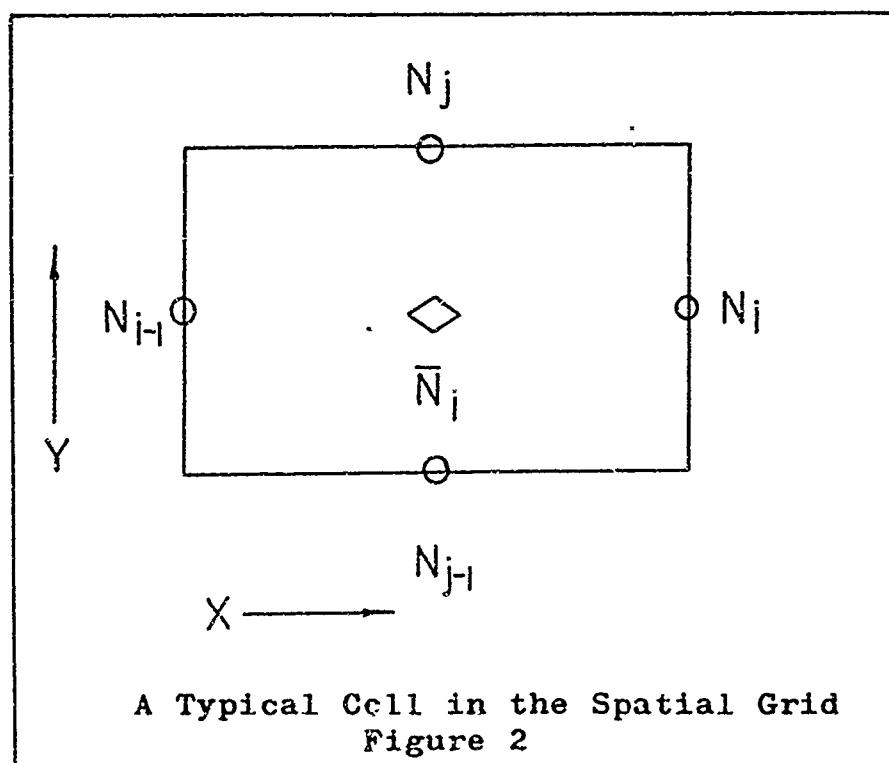
The source term,  $S^g$ , is composed of four quantities; virgin neutrons from fissions, scattered neutrons from high-energy groups, "weak" or within-group scatter, and any external sources. By assuming all scatters to be isotropic this source term is dependent only on position, as was stated earlier. This assumption is reasonable since fission has been found experimentally to be very nearly isotropic, and scattering events are virtually isotropic for atoms with an atomic weight of 10 or greater. There are no external sources in this problem. Operating on this term as before, one has

$$\frac{1}{\Delta X \Delta Y \Delta \mu \Delta \phi} \int_{X_{i-1}}^{X_i} \int_{Y_{j-1}}^{Y_j} \int_{\mu_{n-1}}^{\mu_n} \int_{\phi_{\ell-1}}^{\phi_\ell} [S(X, Y)] = S(\bar{X}_i, \bar{Y}_j) \equiv \bar{S}_i \quad (13)$$

Eq (1) now has the difference form

$$\frac{\eta_n \cos \bar{\phi}_\ell}{\Delta X} \left[ N_i^g - N_{i-1}^g \right] + \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta Y} \left[ N_j^g - N_{j-1}^g \right] + \sum_T \bar{N}_i^g = \bar{S}_i^g \quad (14)$$

Eq (14) is the desired difference equation which relates the five discrete fluxes as shown in Figure 2. One may obtain difference equations relating any three of these discrete fluxes by use of the diamond difference technique.



The diamond difference assumption is that  $2\bar{N}_i = N_i + N_{i-1} = N_j + N_{j-1}$  (Ref 13:68), which says merely that the average of the flux across one cell in any one variable is the average for the whole cell.

One may now use this approximation to eliminate two of the five fluxes in Eq (14). Eliminating  $\bar{N}_i$  and  $N_j$ , one obtains

$$\frac{\eta_n \cos \bar{\phi}_e}{\Delta x} [N_i - N_{i-1}] + \frac{\eta_n \sin \bar{\phi}_e}{\Delta y} [N_i + N_{i-1} - N_j - N_{j-1}] + \frac{1}{2} \sum_T [N_i + N_{i-1}] = \bar{S}_i \quad (15)$$

which may be written

$$\left[ \frac{\eta_n \cos \bar{\phi}_\ell}{\Delta x} + \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta y} + \frac{\sum_T}{2} \right] N_i + \left[ \frac{\eta_n \cos \bar{\phi}_\ell}{\Delta x} + \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta y} + \frac{\sum_T}{2} \right] N_{i-1} + \quad (16)$$

$$\left[ \frac{-2\eta_n \sin \bar{\phi}_\ell}{\Delta y} \right] N_{j+1} = \bar{S}_i$$

Alternately, one may eliminate  $\bar{N}_i$  and  $N_{j-1}$  as follows

$$\frac{\eta_n \cos \bar{\phi}_\ell}{\Delta x} [N_i - N_{i-1}] + \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta y} [N_j - N_i - N_{i-1} + N_j] + \frac{1}{2} \sum_T [N_i + N_{i-1}] = \bar{S}_i \quad (17)$$

which may be written

$$\left[ \frac{\eta_n \cos \bar{\phi}_\ell}{\Delta x} - \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta y} + \frac{\sum_T}{2} \right] N_i + \left[ \frac{-\eta_n \cos \bar{\phi}_\ell}{\Delta x} - \frac{\eta_n \sin \bar{\phi}_\ell}{\Delta y} + \frac{\sum_T}{2} \right] N_{i-1} + \quad (18)$$

$$\left[ \frac{2\eta_n \sin \bar{\phi}_\ell}{\Delta y} \right] N_j = \bar{S}_i$$

Eqs (16) and (18) are the two working equations.

These working equations have been incorporated into a computer program called S4C40 which is described in Appendix A.

The source term  $\bar{S}_i^g$  in these equations is calculated in the following manner (Ref 3:6). From an inspection of Eq (1), it is seen that  $N(x, y, \mu, \phi)$  is the flux per steradian with the units (neutrons/cm<sup>2</sup>-sec-steradian).

The isotropic or all-angle flux is  $\int_{4\pi} N(X, Y, \mu, \phi) d\Omega$

Since the problem is assumed to be symmetric with respect to  $z$ , only one hemisphere need be considered in the calculation. The areas on the unit sphere about each discrete direction are equal (Ref 1), and hence the fluxes so calculated are of equal importance or weight. The all-angle flux  $\bar{N}(\vec{r})$  is thus

$$\bar{N}(\vec{r}) = 2\pi \left[ \frac{2}{n} \sum_{i=1}^n \bar{N}_i(\vec{r}) \right] \quad (19)$$

where  $n$  is the number of discrete directions taken. The source term per steradian is

$$\bar{S}_i(\vec{r}) = \frac{1}{4\pi} \bar{N}(\vec{r}) \quad \bar{N}(\vec{r}) = \frac{1}{n} \sum_{i=1}^n \bar{N}_i(\vec{r}) \quad (20)$$

For an  $S_2$  calculation, as shown in Figure 3,  $N=4$ , and for an  $S_4$  calculation, as shown in Figure 4,  $N=12$ , where  $N$  is the number of discrete directions in space on the hemisphere. For an  $S_n$  problem in xy-geometry, Eq (14) may be written

$$\frac{\eta_m \cos \bar{\phi}_\ell}{\Delta X} [N_i - N_{i-1}] + \frac{\eta_m \sin \bar{\phi}_\ell}{\Delta Y} [N_j - N_{j-1}] + \sum_T \bar{N}_i = \bar{S}_i \quad (21)$$

$$m=1, 2, \dots, \frac{n}{2}; \quad l=\pm 1, \pm 2, \dots, \pm \frac{n}{2} - 1$$

These discrete directions are the quadrature points for the integration over the hemisphere. (For other integration schemes see Refs 12, 13:40, and 15:136.) If one defines a flux

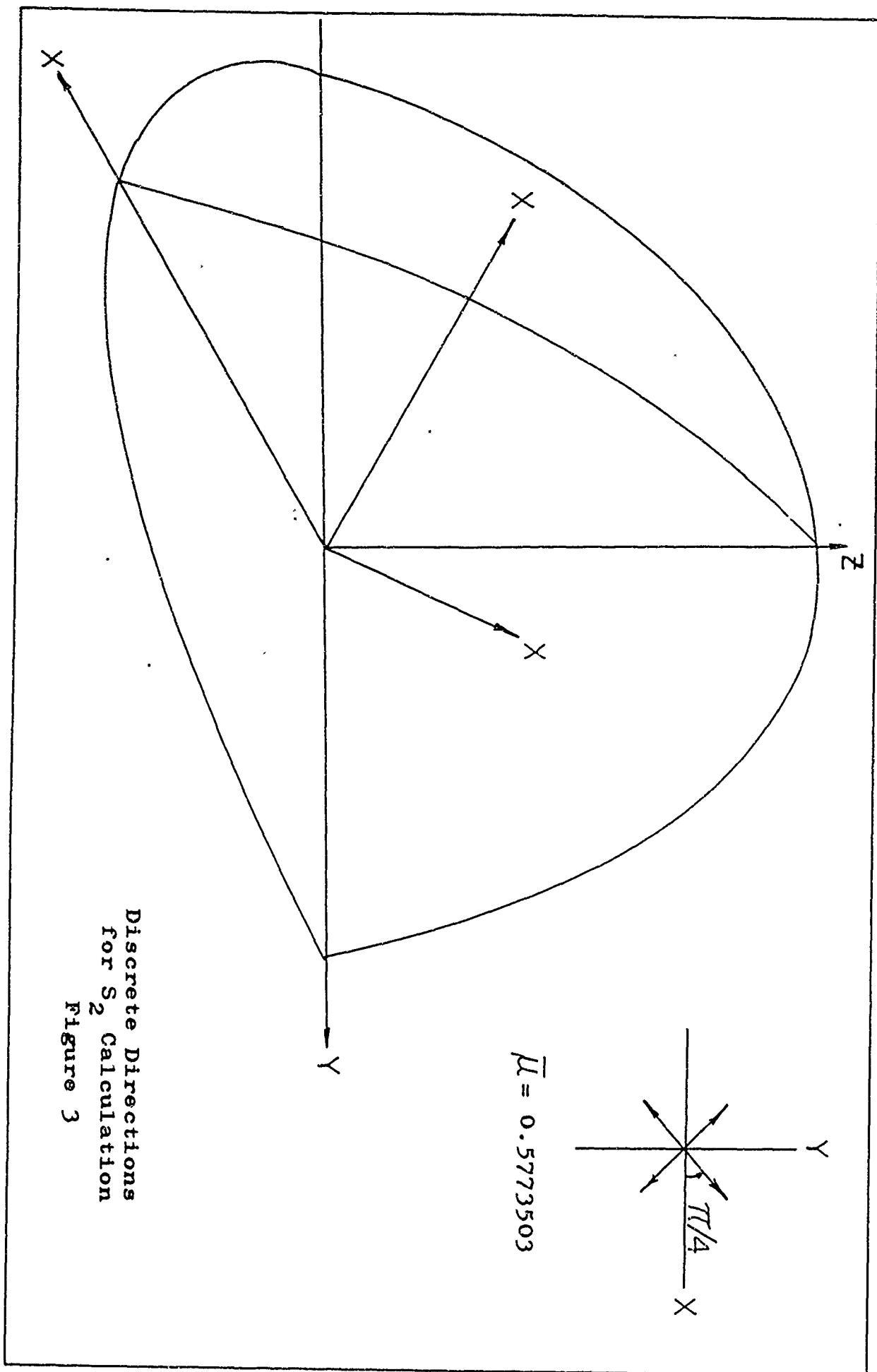
$$\bar{N}'(\vec{r}) = \frac{1}{n} \sum_{i=1}^n \bar{N}_i(\vec{r}) \quad (22)$$

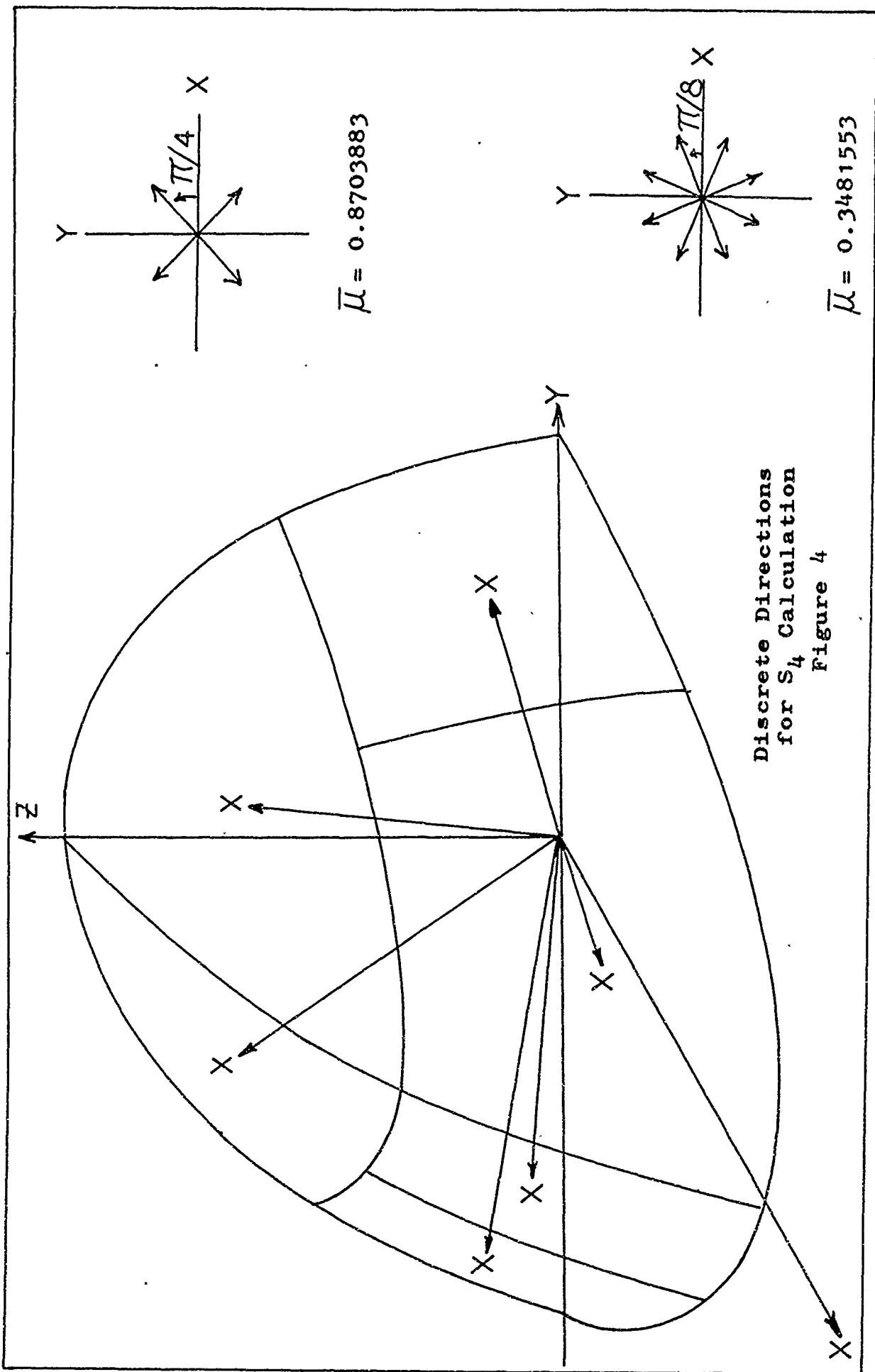
this flux can be put directly into the source term. Note that  $\bar{N}'(\vec{r}) = \frac{1}{4\pi} \bar{N}(\vec{r})$ , differing only by a constant, and hence can be taken as a relative measure of the all-angle flux. The set of  $\bar{N}'(\vec{r})$  for the mesh of discrete points is the end result of the calculations.

The values of  $\mu_m$  and  $\phi_\ell$  are chosen so that equal areas (solid angles) are described on the unit sphere. It remains now to determine the values of  $\bar{\mu}_m$  and  $\bar{\phi}_\ell$  to be used in Eqs (16) and (18). This is done by demanding that the  $S_n$  approximations give the same results as diffusion theory in regions where the latter is applicable (Ref 5:538). In this case the flux is isotropic, and, neglecting the amplitude, the current at a point is given by

$$J(\vec{r}) = 2 \int_0^1 \mu N(\vec{r}, \mu) d\mu \stackrel{n/2}{\approx} \sum_{m=1}^{n/2} w_m \bar{\mu}_m^2 = \frac{1}{3} \quad (23)$$

where the  $w_m$  are the point weights in the numerical integration scheme used. The  $\mu_m$  can be written (for equal areas on the unit sphere, as shown in Figures 3 and 4) as





$$\mu_m = -i + \frac{m}{2Q}(m+1), m=0, 1, 2, \dots, \frac{n}{2} \quad (24)$$

where

$$Q = \sum_{m=1}^{n/2} m = \frac{n}{2} \left( \frac{n}{2} + 1 \right) \frac{1}{2} \quad (25)$$

One assumes a form  $\bar{\mu}_m = \frac{1}{2}(\mu_m + \mu_{m-1})\epsilon$ , where  $\epsilon$  is some constant factor to be determined. The weights are

$$w_m = \frac{1}{2}(\mu_m - \mu_{m-1}) = \frac{m}{2Q} \quad (26)$$

so that

$$\bar{\mu}_m^2 = \epsilon^2 \left( 1 - \frac{m^2}{Q} + \frac{m^4}{4Q^2} \right) \quad (27)$$

Substituting the expressions for  $w_m$  and  $\bar{\mu}_m^2$  into Eq (23) and using the following algebraic identities

$$\sum_{x=i}^j x = \frac{1}{2} j(j+1) \quad (28)$$

$$\sum_{x=i}^j x^3 = \frac{1}{4} j^2(j+1)^2 \quad (29)$$

and

$$\sum_{x=i}^j x^5 = \frac{1}{12} j^2(j+1)^2(2j^2+2j-1) \quad (30)$$

one finds that

$$2 \sum_{m=1}^{n/2} w_m \bar{\mu}_m^2 = \frac{\epsilon^2}{24Q} (n^2 + 2n - 2) = \frac{1}{3} \quad (31)$$

The solution for  $\epsilon$  is

$$\epsilon = \frac{\sqrt{n^2 + 2n}}{\sqrt{n^2 + 2n - 2}} \quad (32)$$

Hence, the general expression for  $\bar{\mu}_m$  is

$$\bar{\mu}_m = \frac{\frac{1}{2}(\mu_m + \mu_{m-1}) \sqrt{n^2 + 2n}}{\sqrt{n^2 + 2n - 2}} \quad (33)$$

These values are tabulated in Table IV of Ref 2:230.

The  $\bar{\phi}_l$  are not weighted but taken to be the appropriate mean values, and are given in Table V of Ref 2:231. The values for this problem are given in Table I below.

Table I  
Values of Angles for an  $S_4$  Calculation  
in XY-Geometry

$m$	$\mu_m$	$\bar{\mu}_m$	$l$	$\bar{\phi}_l$
1	1	0.8703883	1	$\pi/4$
			2	$3\pi/4$
2	0.667	0.3481553	1	$\pi/8$
			2	$3\pi/8$
			3	$5\pi/8$
			4	$7\pi/8$

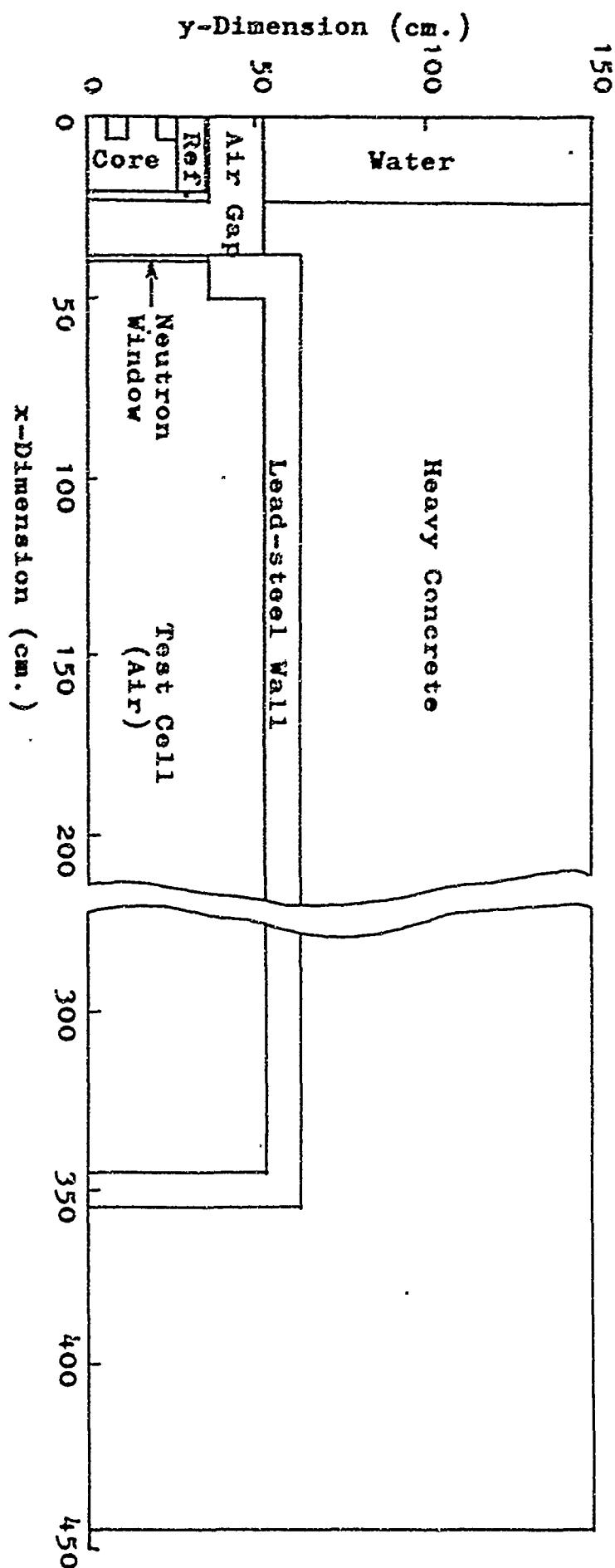
III. Calculational Method

As in all reactor calculations, choosing the constants in the problem and developing a suitable model (the grid for the numerical calculation) are two of the most difficult decisions, mainly because so many choices are possible. This chapter includes the model used for the NETR and the method of generating cross-sections for S4C40.

S4C40 Calculational Grid

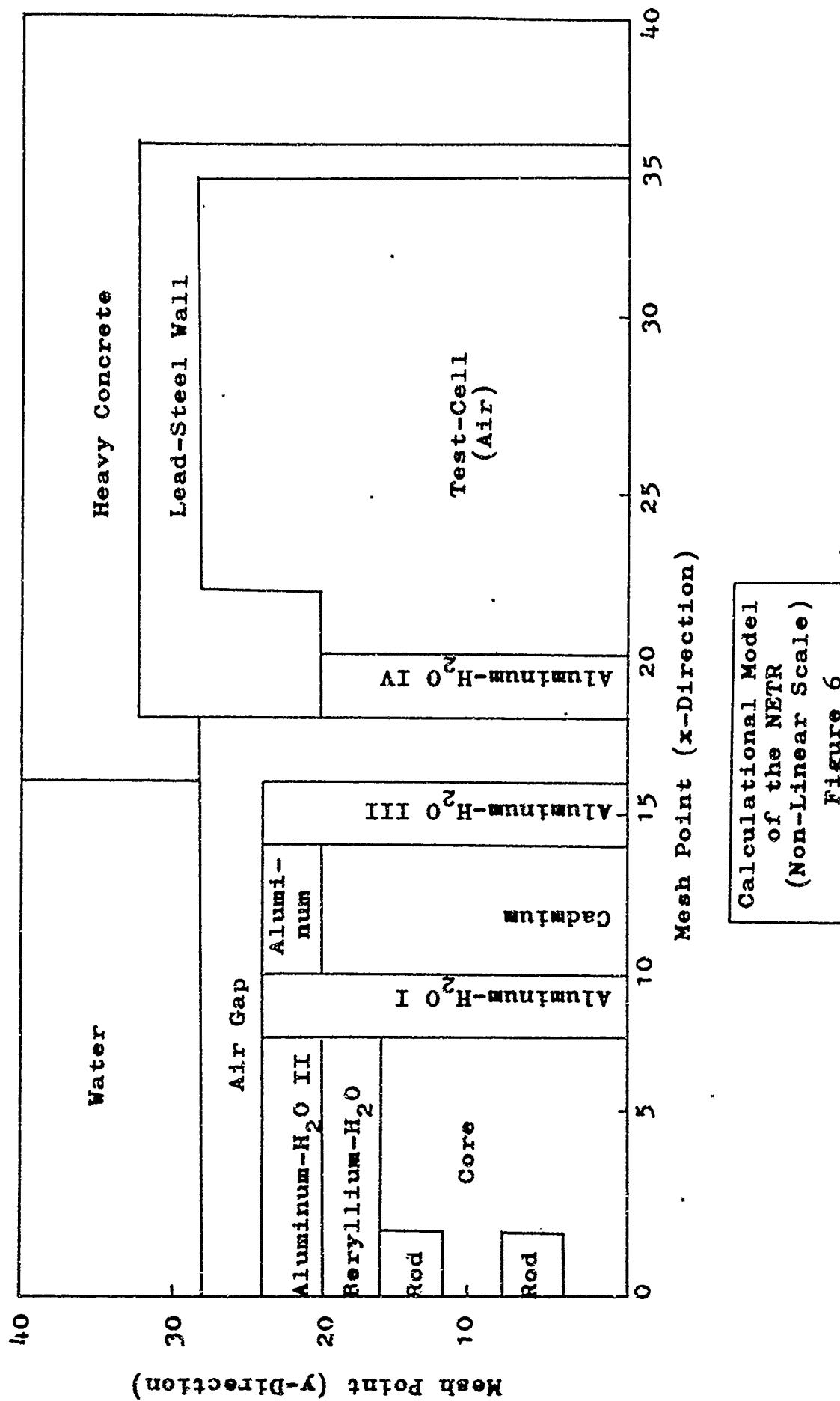
A sketch of the NETR is shown in Figure 1. Note the presence of water on both the north and south faces of the reactor, of the test-cells on the east and west sides, and of the cadmium slabs on the faces of the core nearest each test-cell to reduce the thermal flux. Heavy (barytes) concrete surrounds the test-cells for biological shielding. As Figure 5 illustrates, these features have been incorporated in the grid. It should be noted that this problem is inherently two-dimensional in character.

The final grid as shown in Figure 6 was based on that of Garrett and Peñaranda (Ref 8:25) with some modification as noted. Fewer points have been taken in the core and in the concrete, and the aluminum and water regions surrounding the core have been homogenized. Approximately the same number of points has been used in the test-cell (15 vs 17 in the x-direction). These modifications were made to keep the computational time



Calculation Model  
of the NETR  
(Linear Scale)

Figure 5



to a minimum, consistent with acceptable accuracy.

The thermal flux in the interior of the cadmium curtain was set identically equal to zero. This can be viewed as an additional "boundary condition" imposed on the problem and was done primarily to suppress oscillations in that region and to aid convergence. Carlson predicts (Ref 2:222) that when the product of the cross-section and the spatial increment is large, oscillations in the calculated fluxes may occur. This was found to happen both in the cadmium, and in the more removed parts of the test-cell when an increment of 37 centimeters was taken. When this increment was reduced to 22.5 centimeters the values were more stable, although some oscillation remained.

#### Cross-sections

The fast and epithermal cross-sections used in this calculation were calculated with the GAM-I code (Refs 10, 11) and the thermal cross-sections were computed by hand assuming a Maxwell-Boltzmann distribution and a temperature of 58.5° C. (See Appendix B.)

The energy limits were taken to be the same as those used by Garrett and Peñaranda (Ref 8) for purposes of comparison with PDQ (Ref 1:781-783, 787). Atom densities of the various materials were taken from manufacturer's data; the density of air was computed at a temperature of 20° C.

Input Flux

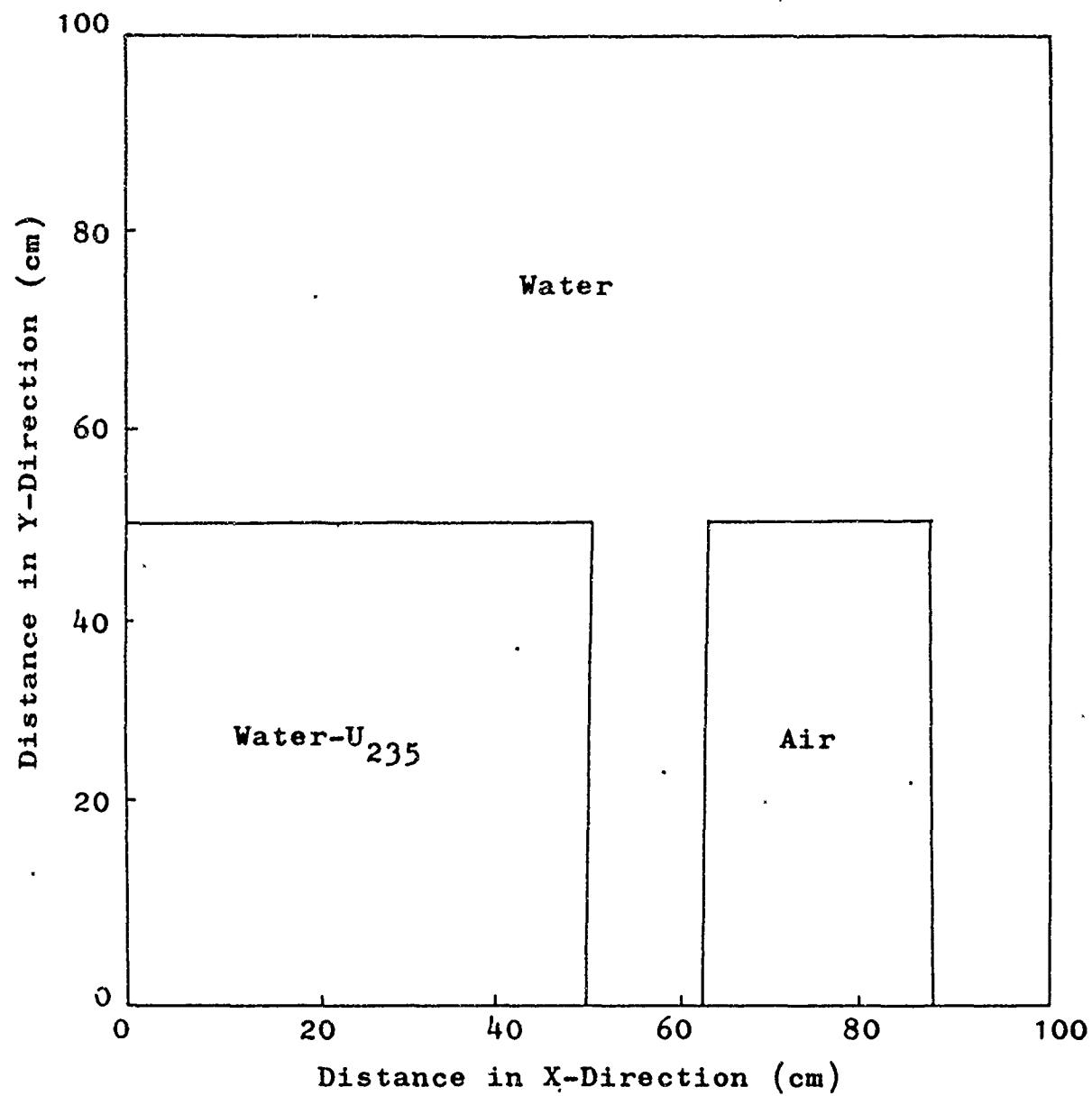
The initial estimate of the flux was calculated using the  $S_2$  approximation and a 20x20 mesh. After the S4C40 code was first run, the last computed values of flux were used as input for the succeeding run. This allowed the problem to be stopped and restarted at will.

IV. Results

The sophistication of the equations in the last chapter and the complexity of their coupling required that a great deal of effort and time (about 2½ months) be spent in debugging the computer code. The correspondence in Appendix C refers to five of the many problems which arose in this work. This correspondence has been included in the hope that it may prove useful to others in future work. A test problem, which is described below, was employed during this debugging to gauge the output of the program.

Test Problem

The purpose of the test problem was to determine whether the errors in Fortran programming had been eliminated and to judge whether the output of the program was reasonable. The test problem was an  $S_2$  approximation, using first a water- $U_{235}$  core surrounded completely by water, and then with an air gap in the water reflector parallel to one face of the core as shown in Figure 7. The atom density of fuel and water in the core was calculated using modified one-group theory. Various grid spacings were employed, the final one being ten points in the y-direction 10 cm. apart, and sixteen points in the x-direction 6.25 cm. apart. The initial flux estimate was made intentionally poor to determine if the



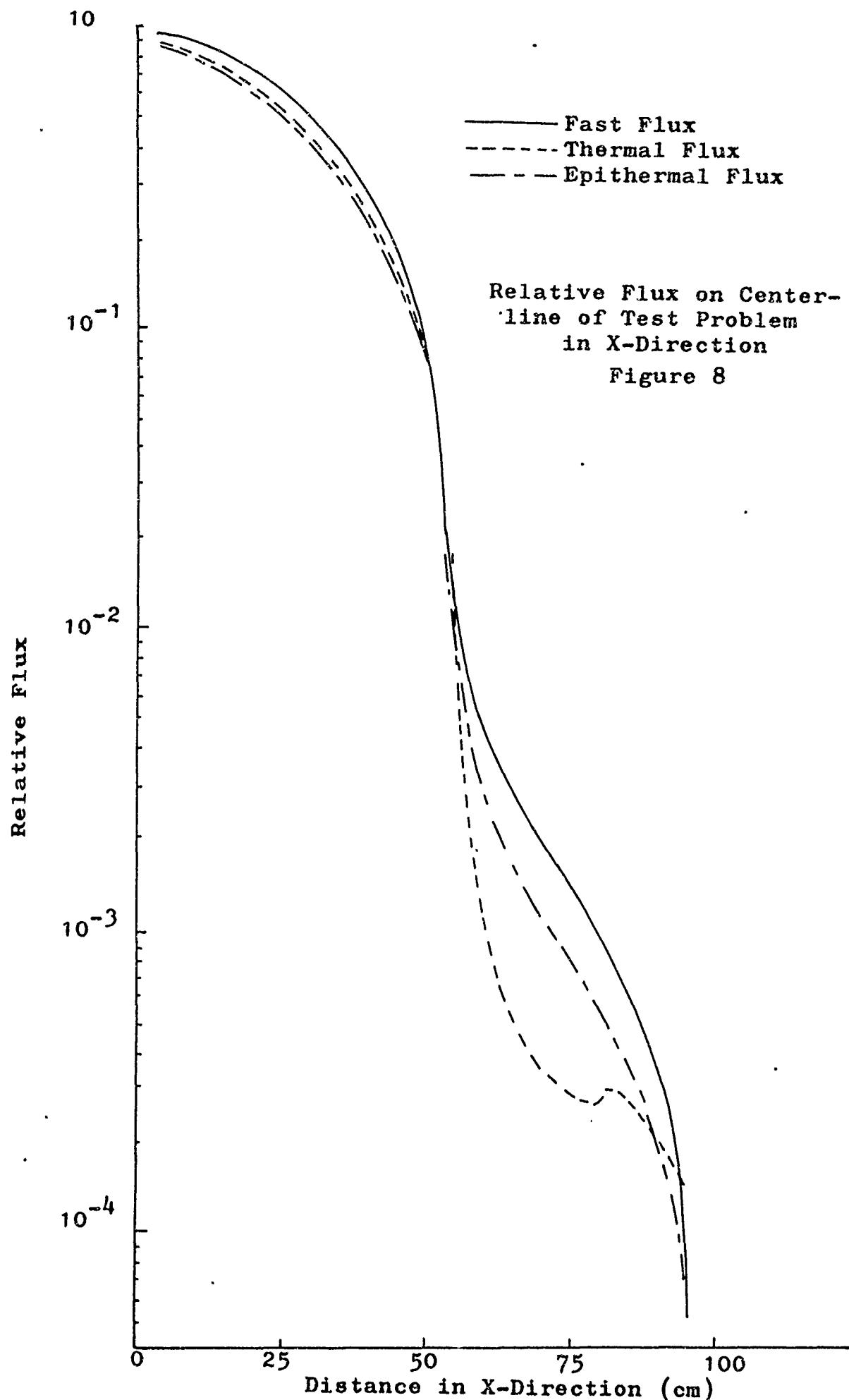
Calculational Grid  
for Test Problem  
Figure 7

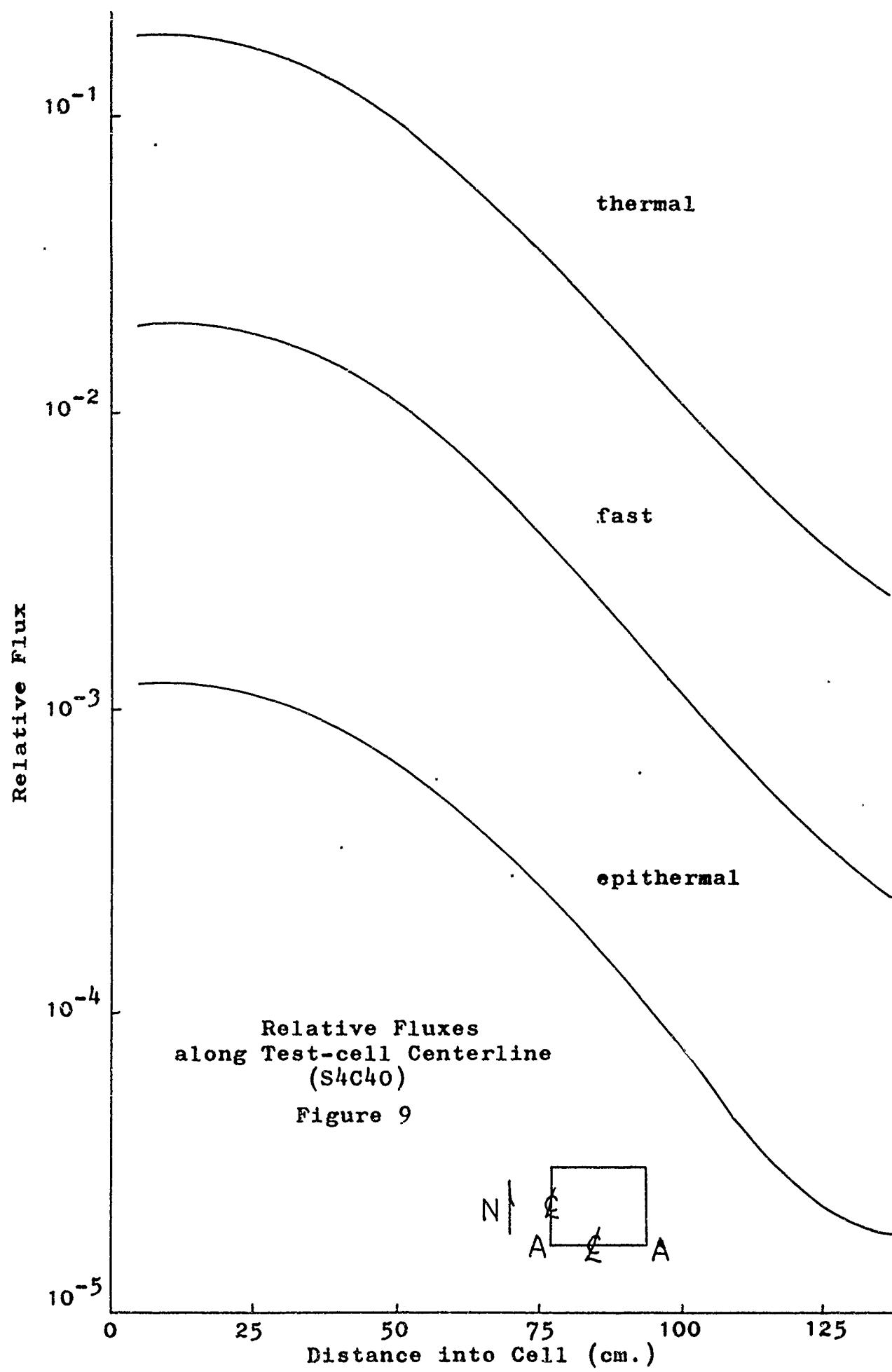
program were sensitive to these input values. It was found that the initial flux strongly influenced the calculated values, the flux shape changing very slowly. One-fourth the reactor was considered, using symmetry along the centerlines in both the x- and y-directions.

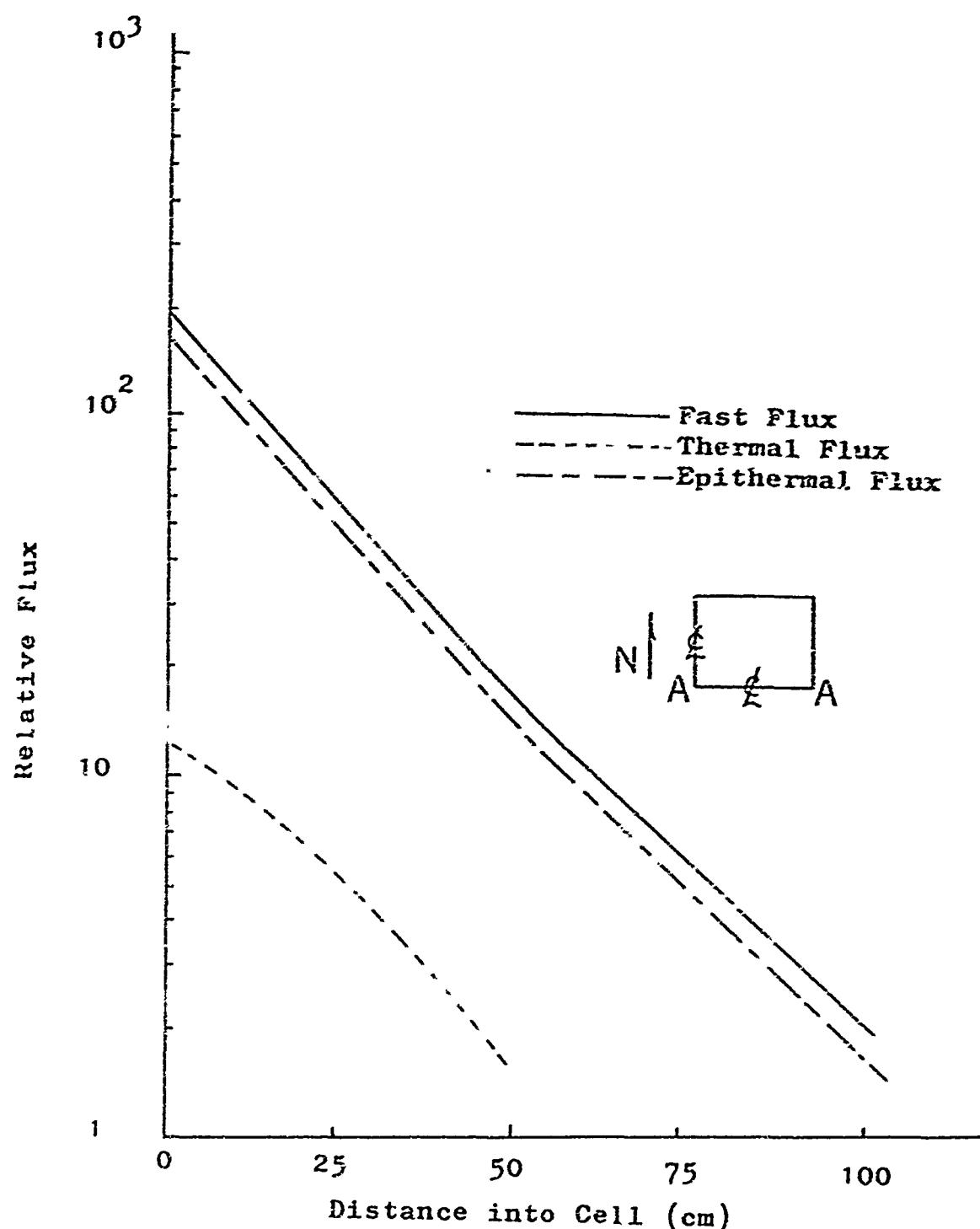
The calculated flux along the x-centerline is shown in Figure 8. These values were computed after 176 iterations. The thermal flux is somewhat high in the core, being approximately equal to the fast flux. The values in the air gap look very good, the higher-energy fluxes dropping almost exponentially and the thermal flux dropping initially and then rising because of backscatter and thermalization. A two-group, one dimensional diffusion theory hand calculation gave nearly the same results.

#### NETR Results

The final flux calculated along the centerline of the test-cell is shown in Figure 9; for comparison, the flux calculated by PDQ is shown in Figure 10. (The small insert in Figures 9-13 shows the location in the test-cell of the abscissa of the figure.) The fast and epithermal fluxes are in fairly good agreement, although the epithermal flux is noticeably lower in magnitude. The thermal flux predicted by S4C40 is two orders of magnitude greater than that predicted by PDQ; this is true not only in the test-cell but also for the entire reactor. The thermal flux was everywhere decreased by a







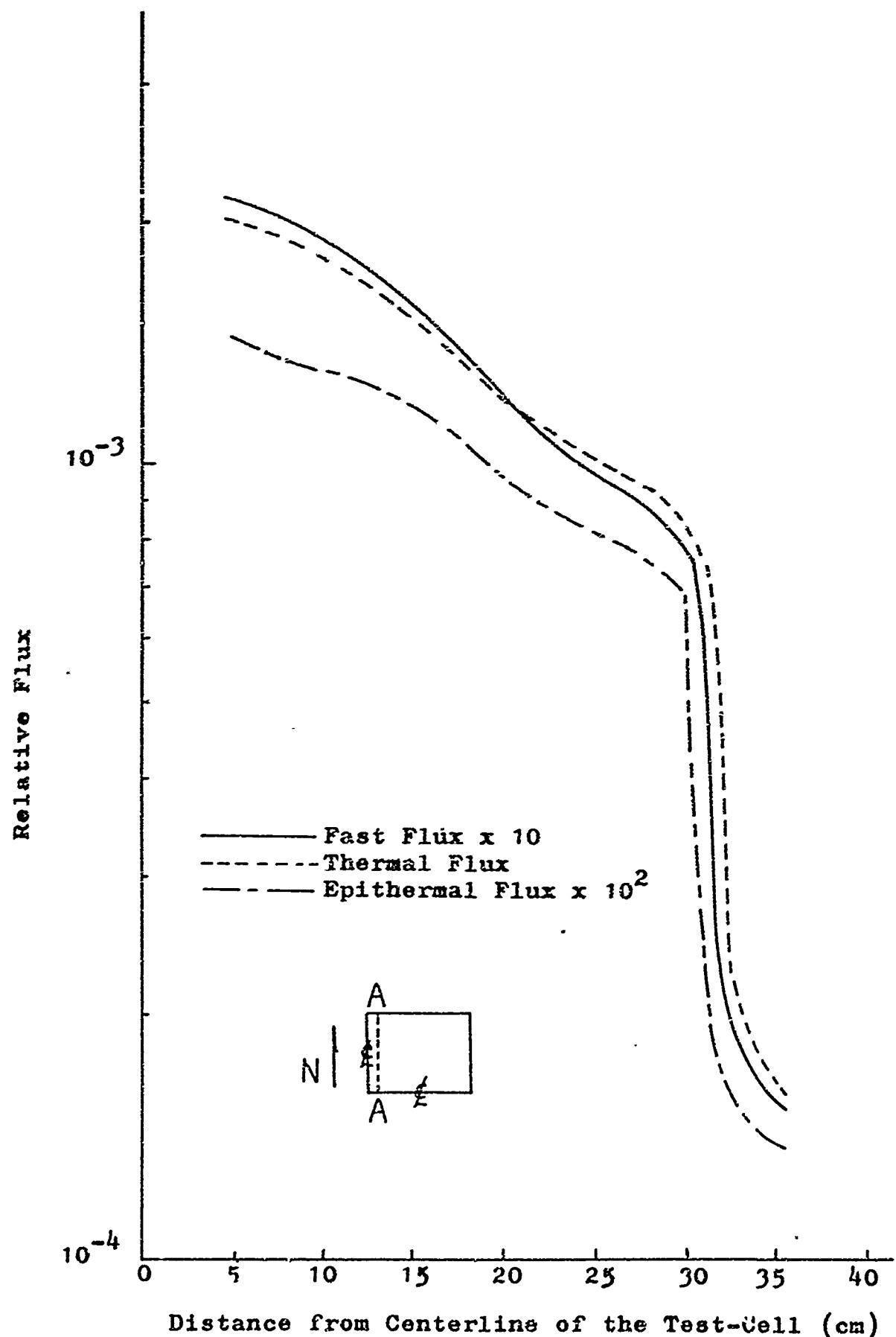
Relative Fluxes  
along Test-Cell Centerline  
(PDQ)

Figure 10

factor of 100 and S4C40 executed again to see if this were the result of a poor initial estimate. After five iterations, the program calculated essentially the original values. Due to time limitations, the solution of this problem must be left to other investigators.

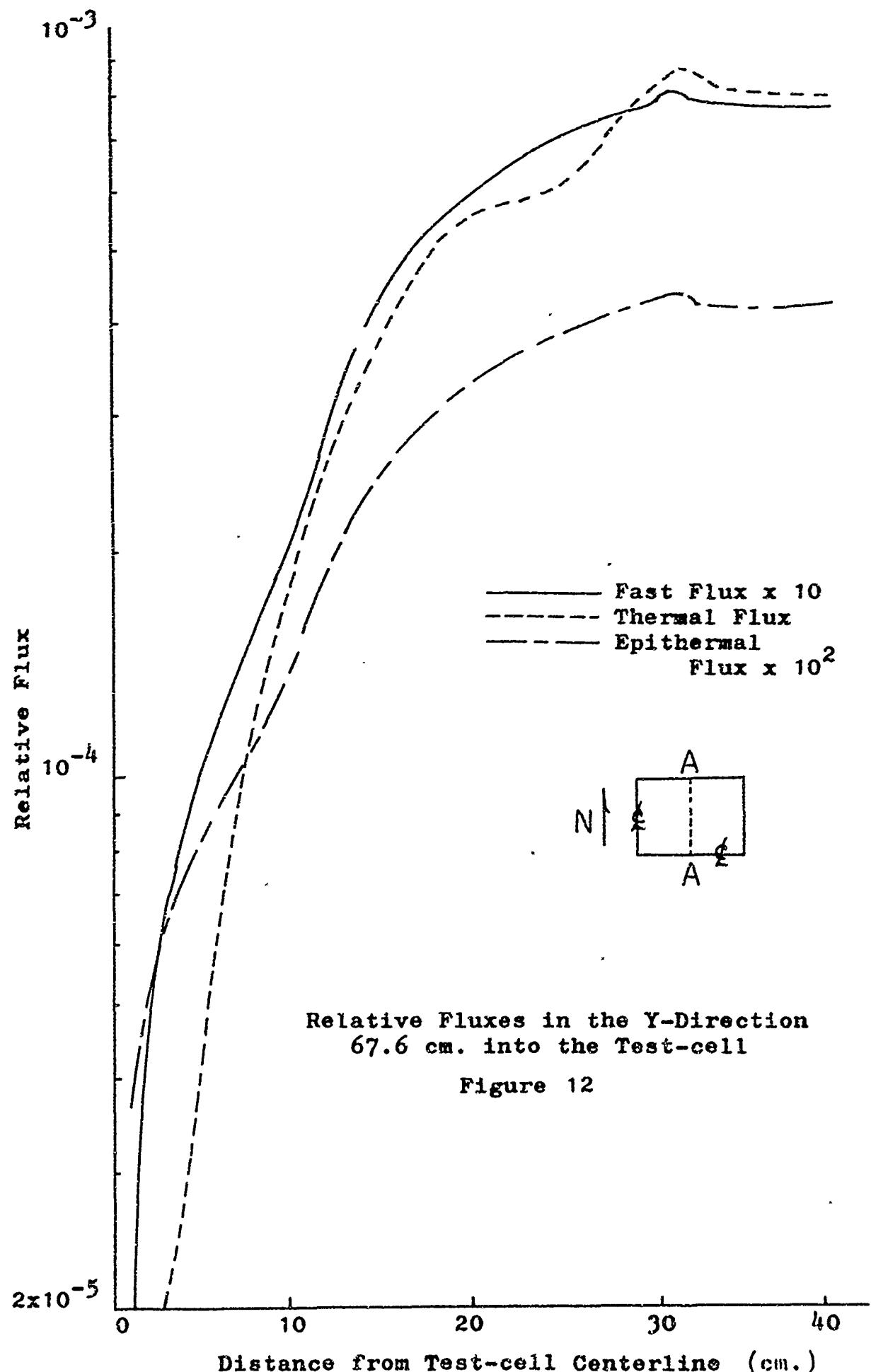
S4C40 predicts a flux decrease of one order of magnitude in 100 centimeters in the test-cell, whereas PDQ predicts a drop of three orders of magnitude in the same distance. This indicates that a higher flux is available for experiments than had been thought previously.

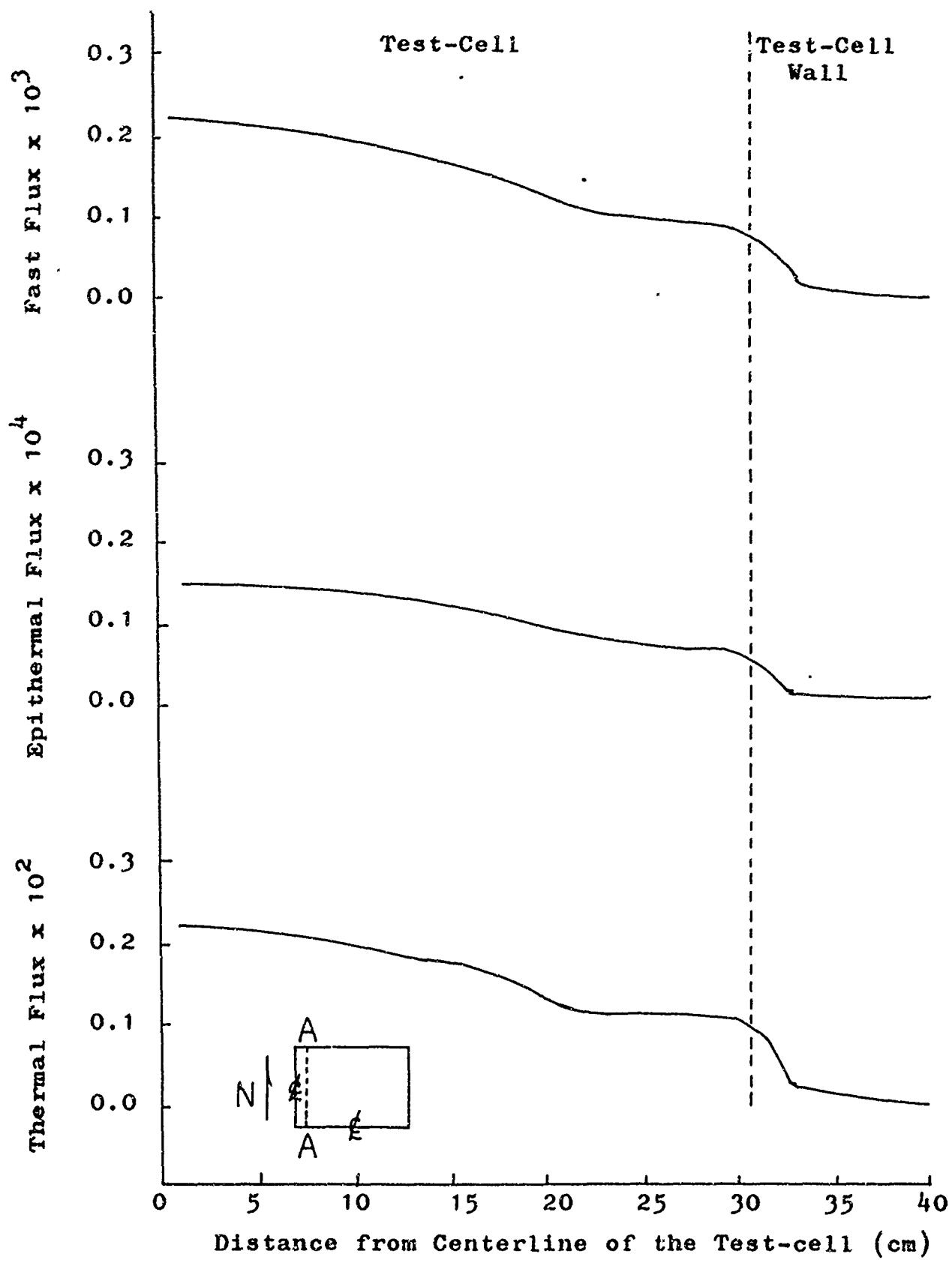
The flux across the test-cell, for two distances in the x-direction, is shown in Figures 11 and 12. The relative magnitudes of the different energy fluxes remains essentially constant. Near the neutron window the flux drops off slowly as one moves away from the centerline. At a distance of 67.6 centimeters the reverse is true, a minimum occurring near the centerline nearly two orders of magnitude lower than near the sides of the test-cell. This point is discussed further in Chapter V. The dip in the flux shown in Figure 12 at about 31 centimeters is probably due to shadow-shielding by the test-cell wall. (See Figure 5.) The semi-logarithmic plot in Figures 11 and 12 is misleading. The derivative of the flux at the centerline in both cases is actually zero. Figure 11 has been plotted on a linear scale in Figure 13 to demonstrate this point.



Relative Fluxes in the Y-Direction  
2.83 cm into the Test-Cell

Figure 11





Relative Fluxes in the Y-Direction  
2.83 cm into the Test-cell

Figure 13

#### V. Conclusions and Recommendations

The discussion in Chapter IV suggests that the transport equation, although basically the proper approach to the problem, predicts fluxes which are, in this case, dubious at best. If the flux predicted by the program is correct, one possible explanation for the high thermal flux in the test-cell is backscatter. The high-energy neutrons emerging from the core may become thermalized in the water and concrete and subsequently diffuse back into the test-cell, thus defeating the purpose of the cadmium curtain. This hypothesis could be tested by executing S4C40 with a cadmium lining around the test-cell. (Appendix A)

The  $S_n$  approximation is easily programmed but is rather slow to converge. Carlson predicts that about 200 iterations are required for convergence for a thermal reactor (Ref 5:538); since S4C40 requires one minute six seconds per iteration, this would mean about three hours of computing time to achieve convergence.

There are a number of refinements which may be included in the program to increase the accuracy of the results. Four will be mentioned briefly. The most significant improvement would be obtaining more accurate cross-sections. In the core region, a separate transport calculation could be performed to predict fission cross-sections which take into account the thermal flux depression in the fuel. Total and scattering cross-

sections in the core could be similarly calculated, permitting the homogenized core to better reflect reality. In addition it should be advantageous to divide the energy spectrum into more groups. Table II gives a suggested set of energy groups over the range of 10 Mev to 0.414 ev. These groups were selected to reflect the resonance regions of the predominant elements present in the reactor. It may also be necessary to consider group structure within the thermal region. Such data could be generated by a program such as SLOP-1 (Ref 1:745).

A second refinement would be to consider anisotropic scatter, especially in the test-cell where the relatively light elements oxygen and nitrogen are present. Both Carlson (Ref 3:19) and Lee (Ref 13:110) discuss techniques of handling this.

When S4C40 calculates a negative angular flux, the flux is set to zero and the computation continued. A better technique would be to adjust the angular flux on the opposite edge of the cell and the average flux, as well as setting the negative quantity to zero (Ref 4:37).

Still a fourth refinement would be to adjust the model in any of several ways. Since the NETR is not exactly symmetric, as shown in Figure 1, a model using half the reactor would be more realistic (Ref 8:26,27). Alternately, more mesh points in the  $\frac{1}{4}$ -core model, which is shown in Figure 6, or a three-dimensional model using five or so points in the z-direction would improve accuracy.

Table II  
Cross-section Group Structure

Group	Lower Energy (ev)	$\Delta U$	Lower GAM-I Group
1	$10^7 - 7.79 \times 10^6$	0.25	1
2	$3.68 \times 10^6$	0.75	4
3	$4.98 \times 10^5$	2.00	12
4	$3.02 \times 10^5$	0.50	14
5	$3.18 \times 10^4$	2.25	23
6	$9.61 \times 10^2$	3.50	37
7	$4.54 \times 10^2$	0.75	40
8	$1.37 \times 10$	3.50	54
9	5.04	1.00	58
10	0.41	2.50	68

Any of the refinements listed above should improve the accuracy of the code and, with sufficient iterations, any desired degree of precision could be achieved.

## Bibliography

1. Argonne National Laboratory. Reactor Physics Constants. USAEC Document ANL-5800. Argonne, Illinois, July 1963.
2. Carlson, B. G., "Numerical Solution of Neutron Transport Problems." Proceedings of Symposia in Applied Mathematics, 11: 219-232 (1959).
3. -----. Solution of the Transport Equation by S<sub>n</sub> Approximations. LA-1891. Los Alamos Scientific Laboratory, February 1955.
4. -----. Numerical Formulation and Solution of Neutron Transport Problems. LA-2996. Los Alamos Scientific Laboratory, April 1964.
5. Carlson, B. G., and G. I. Bell. "Solution of the Transport Equation by the S<sub>n</sub> Method." Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, 16: P/2386. 535-549 (1958).
6. Case, K. M., F. de Hoffman, and G. Placzek. Introduction to the Theory of Neutron Diffusion, 1. Washington: US Government Printing Office, 1953.
7. Cranberg, L., G. Frye, N. Nereson, and L. Rosen. "Fission Neutron Spectrum of U-235." Physical Review, 103: 662 (1956).
8. Garrett, C. W., and F. E. Peñaranda. The AF-NETR Reactor Physics Calculations, 1956-1964. Wright-Patterson Air Force Base, Ohio: Air Force Flight Dynamics Laboratory, September 1964.
9. Hurwitz, H., and P. F. Zweifel. "Slowing Down of Neutrons by Hydrogenous Moderators." Journal of Applied Physics, 26: 923 (1955).
10. Joanou, G. D., and J. S. Dudek. GAM-I: A Consistent P<sub>1</sub> Multigroup Code for the Calculation of Fast Neutron Spectra and Multigroup Constants. USAEC Document GA-2451. San Diego, California: John Jay Hopkins Laboratory for Pure and Applied Science, 1961.
11. Joanou, G. D., et al. Nuclear Data for GAM-I Data Tape, Volumes I and II. USAEC Document GA-2451. San Diego, California: John Jay Hopkins Laboratory for Pure and Applied Science, 1961.

12. Lathrop, K. D., and B. G. Carlson. Discrete Ordinates Angular Quadrature of the Neutron Transport Equation. LA-3186. Los Alamos Scientific Laboratory, September 21, 1964.
13. Lee, C. E. Discrete Sn Approximation to Transport Theory. LA-2595. Los Alamos Scientific Laboratory, June 1961.
14. Meghrebian, R. V., and D. K. Holmes. Reactor Analysis. New York: McGraw-Hill Book Company, Inc., 1960.
15. Scarborough, J. B. Numerical Mathematical Analysis (Fifth Edition). Baltimore: The Johns Hopkins Press, 1962.
16. Weinberg, A. M., and E. P. Wigner. The Physical Theory of Neutron Chain Reactors. Chicago: University of Chicago Press, 1958.
17. Yiftah, S., D. Okrent, and P. A. Moldauer. Fast Reactor Cross Sections. New York: Pergamon Press, 1960.

## Appendix A

The S4C40 CodeGeneral Comments

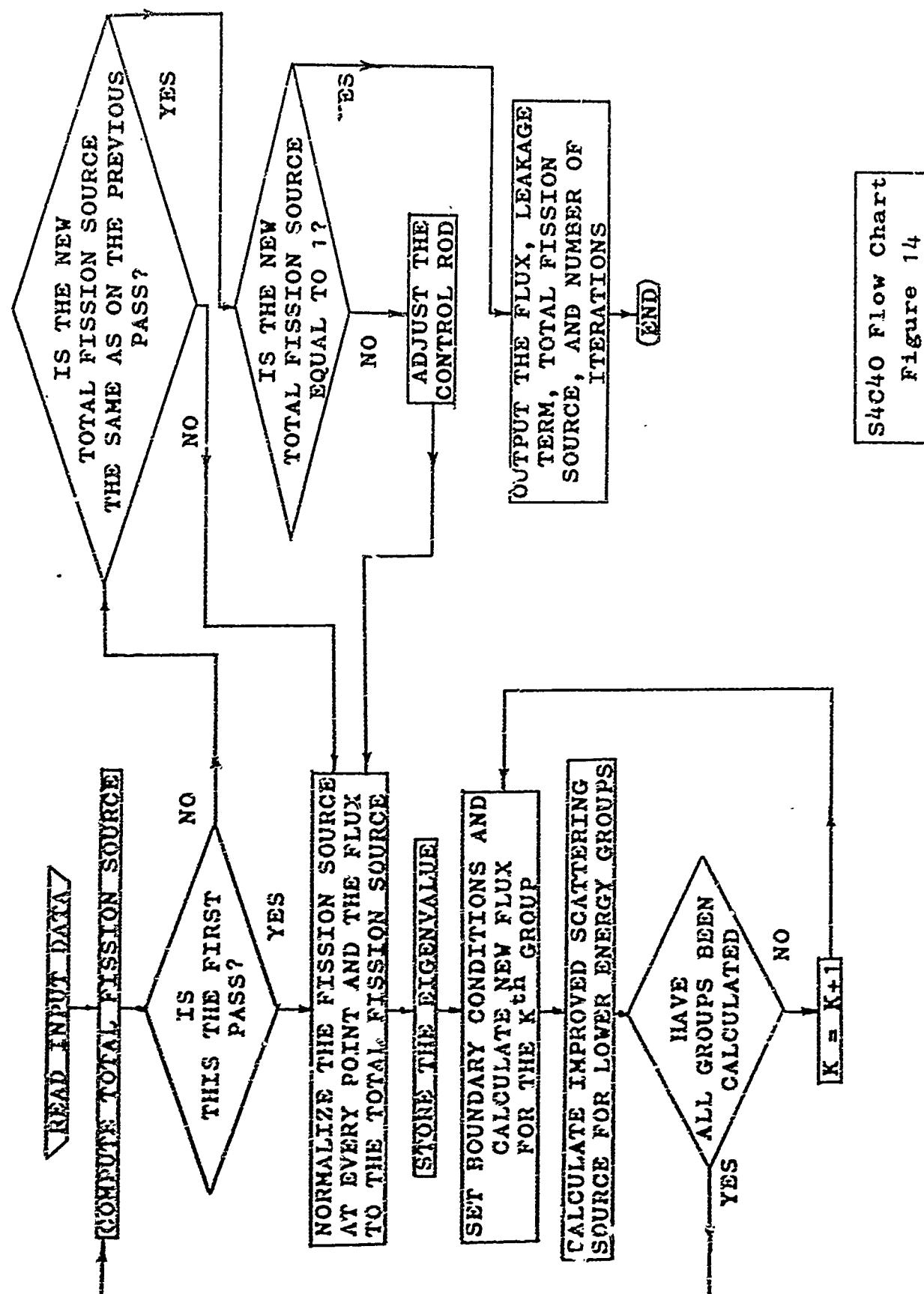
The S4C40 code was written for a particular engineering application and was not designed for use with any other problems. It can be modified easily, however, to incorporate the suggestions mentioned in Chapter V. The program is written to accomodate three energy groups and 1600 mesh points, but these limits may be changed very simply. Since there are many operations which are performed repeatedly in the program, maximum use has been made of subroutines.

All required input data is read in at one time; both comment cards and the sample problem indicate the data needed and the required format. Figure 14 gives a flow chart of the main program. A sample input data, a glossary of variables, and a sample output are included at the end of this appendix.

Subroutines

The following subroutines are called for in the program. Other subroutines, such as computation of parameters allowing for anisotropic scatter, may be added as desired.

Subroutine CON2. This subroutine computes the constants used in Eqs (16) and (18). If one defines the following variables



S4C40 Flow Chart  
Figure 14

$$\eta \cos \bar{\phi}_L / \Delta_i \equiv A_1$$

$$\eta \sin \bar{\phi}_L / \Delta_j \equiv A_2$$

$$(\sum_T \Delta B^2) / 2 \equiv A_3$$

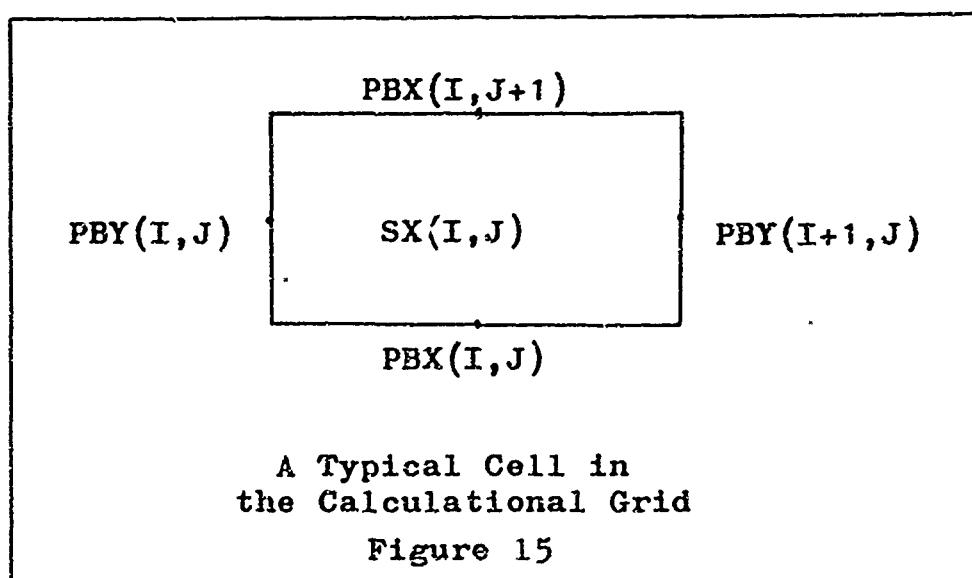
then Eqs (16) and (18) become

$$\begin{aligned}
 & (A_1 + A_2 + A_3) PBY(I+1, J) + (A_1 + A_2 + A_3) PBY(I, J) + (-2A_2) PBX(I, J) \\
 & = SX(I, J) \tag{33}
 \end{aligned}$$

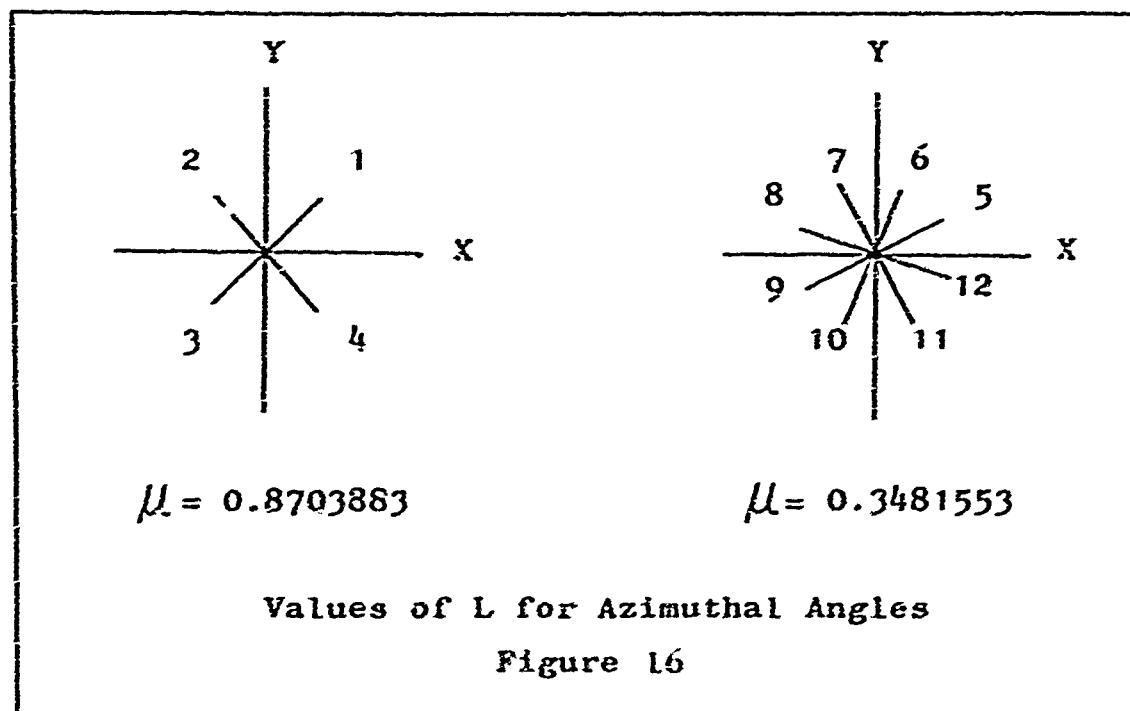
and

$$\begin{aligned}
 & (-A_1 + A_2 + A_3) PBY(I, J) + (A_1 - A_2 + A_3) PBY(I+1, J) + (2A_2) PBX(I, J+1) \\
 & = SX(I, J) \tag{34}
 \end{aligned}$$

where the variables are shown in Figure 15.



The index  $L$ , which is used to compute  $\bar{\phi}_L$ , indicates the angular position as shown in Figure 16.



The order in which the fluxes are calculated is:

$L = 9, 10, 11, 12, 3, 4, 8, 7, 6, 5, 2, 1.$

Subroutine FIS. This subroutine determines whether to use the fission cross-sections for fixed fuel or rod fuel in computing the fission source term SUM for each cell in the core.

Subroutine SCAT. To avoid the necessity of defining the scattering cross-section at each point in the grid and of storing all this data in memory, this subroutine uses the indices I and J of the point to locate the type of material associated with the point. The appropriate cross-sections are then stored in the dummy matrix SCC to be used in the main program.

Subroutine TOTAL. This subroutine duplicates the function of subroutine SCAT, except it stores the total cross-section for the point, for the particular energy group the main program is using, in the location TOT.

Subroutine CKZ. In this subroutine the angular fluxes are set to zero if negative and are accumulated (integrated) in the matrix P4.

Subroutine ROD. In order to drive the reactor critical, some parameter must be variable in the problem. This was accomplished by changing the volume fractions of fuel and cadmium in the rod fuel regions. This subroutine increases the amount of poison if the reactor is supercritical ( $k_{eff} > 1$ ) and vice versa. The efficacy of this technique is doubtful, since the thermal flux was greatly depressed in these regions but the reactor remained supercritical.

Subroutines INOUT7 and INOUT8. These two subroutines allow one to read and write binary information on tapes 7 and 8 respectively. This is more efficient than using decimal numbers, thus decreasing computational time.

```

C THIS PROGRAM REQUIRES SUBROUTINES TOTAL,CKZ,FIS,SCAT,CON2,INOUT7,
C ROD, AND INOUT8
C 2-D, S4 CALCULATION USING CARLSON'S MU(M)-BARS, FOR 40 X 40 MESH
C DIMENSION FS(2,3),FCS(3),S(13,6),ST(13,3),P(40,40,3),
C 1SUM(8,16),V(3),SX(40,40,3),SCC(6),PBX(40,41),PBY(41,40),X(3),P4(40
C 2,40),P5(40,40),DX(40),DY(40),F1(40),F2(40),F3(40),F4(40),F5(40),F6
C 3(40),BXCX(12)
C DIMENSION QC(6),QF(6),QF1(3),QF2(3),QC2(3)
C READ CROSS-SECTIONS, INCREMENTS, VALUES OF NU
C INCREMENTS IN THE X-DIRECTION
C READ(5,105)(DX(I),I=1,40)
C INCREMENTS IN THE Y-DIRECTION
C EVERY OTHER INCREMENT IS READ, EQUAL TO THE
C QUANTITY (DY(I)+DY(I+1)). THESE ARE THEN DIVIDED BY
C TWO AND STORED APPROPRIATELY. IF UNEQUAL SPACING
C IS DESIRED, THIS MAY BE CHANGED TO READ EACH
C OF THE 40 INCREMENTS.
C READ(5,105)(DY(I),I=1,39,2)
C DO 6000 I=2,40,2
C DY(I-1)=DY(I-1)/2.
C 6000 DY(I)=DY(I-1)
C FISSION CROSS-SECTIONS
C FS(1,K) CONTAINS ROD FUEL FISSION CROSS-SECTIONS.
C FS(2,K) CONTAINS FIXED FUEL CROSS-SECTIONS.
C READ(5,106)((FS(M,K),K=1,3),M=1,2)
C VALUES OF NU
C READ(5,106)(V(I),I=1,3)
C VALUES OF CHI
C READ(5,106)(X(I),I=1,3)
C CONVERGENCE CRITERION (EPSILON).
C READ(5,106)AAA
C GROUP TO GROUP SCATTERING CROSS-SECTIONS
C READ(5,105)((S(M,N),N=1,6),M=1,13)
C TOTAL CROSS-SECTIONS
C READ(5,106)((ST(M,K),K=1,3),M=1,13)
C INITIALIZE THE FLUX
C L IS THE NUMBER OF THE UNIT ON WHICH THE
C INITIAL FLUX IS WRITTEN.
C READ(5,99)L
C NUM=0
C READ(5,120)(BXCX(I),I=1,12)
C WRITE(6,119)(BXCX(I),I=1,12)
C READ (L)II,IJ,IK,(((P(I,J,K),I=1,II),J=1,IJ),K=1,IK)
C REWIND L
C DATA NSC,VU,VC/0,1.,0./
C DB2 IS THE PSEUDO-ABSORPTION CROSS-SECTION TO
C ALLOW FOR VERTICAL LEAKAGE.
C DATA NTP,NN,NT,KTR,PI,DB2/4*0,.01745329,.0047752/
C DO 5005 N=1,6

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```

      QC(N)=S(5,N)
5005  QF(N)=S(9,N)
      DO 5006 K=1,3
      QF1(K)=FS(1,K)
      QF2(K)=ST(9,K)
5006  QC2(K)=ST(5,K)
C      SET THE THERMAL FLUX IN THE CENTER OF THE CADMIUM
C      CURTAIN TO ZERO.
      DO 1234 J=1,19
      P(12,J,3)=.0
1234  P(13,J,3)=.0
C      CALCULATE SOURCE TERMS
5      SCE=0.
      NTP=NTP+1
      NN=NN+1
C      CALCULATE THE TOTAL FISSION SOURCE
      DO 2 J=1,16
      DO 2 I=1,8
      CALL FIS(I,J,FS,FCS)
      SUM(I,J)=0.
      DO 3 K=1,3
3      SUM(I,J)=SUM(I,J)+V(K)*FCS(K)*P(I,J,K)
2      SCE=SCE+SUM(I,J)*DX(I)*DY(J)
      IF(KTR)35,35,6
C      NORMALIZE THE FISSION SOURCE
35     DO 4 I=1,8
      DO 4 J=1,16
4      SUM(I,J)=SUM(I,J)/SCE
C      NORMALIZE THE FLUX.
671    DO 950 K=1,3
      DO 950 I=1,40
      DO 950 J=1,40
950    P(I,J,K)=P(I,J,K)/SCE
C      STORE THE EIGENVALUE LAMBDA
      S1=SCE
C      CALCULATE THE SCATTERING SOURCE
      DO 40 J=1,40
      DO 40 I=1,40
      CALL SCAT(S,SCC,I,J)
      DO 40 K=1,3
      SX(I,J,K)=0.
      DO 40 KU=1,K
      N=KU+K-(1/KU)
40     SX(I,J,K)=P(I,J,KU)*SCC(N)+SX(I,J,K)
C      CALCULATE THE TOTAL SOURCE IN THE CORE FOR EACH CELL.
      DO 41 K=1,3
      DO 41 J=1,16
      DO 41 I=1,8
41     SX(I,J,K)=SX(I,J,K)+X(K)*SUM(I,J)

```

```

C BEGIN INNER ITERATIONS
DO 32 K=1,3
C SET BOUNDARY CONDITIONS AT ZERO
DO 7 I=1,40
PBX(I,41)=0.
7 PBY(41,I)=0.
C L IS THE INDEX FOR THE AZIMUTHAL ANGLE PHI
L=9
DO 45 JD=1,40
J=41-JD
DO 45 ID=1,40
I=41-ID
C THE TOTAL CROSS-SECTIONS FOR THE POINT(I,J) ARE DENOTED BY TOT(K)
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
PBY(I,J)=SX(I,J,K)-A*PBY(I+1,J)
PBY(I,J)=(PBY(I,J)-C*PBX(I,J+1))/B
P4(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
45 PBX(I,J)=2.*P4(I,J)-PBX(I,J+1)
C SET ANY NEGATIVE FLUX TO ZERO.
DO 1000 I=1,40
DO 1000 J=1,40
IF(P4(I,J))1001,1000,1000
1001 P4(I,J)=0.
1000 CONTINUE
C THE MATRICES F ARE USED TO STORE THOSE ANGULAR
C FLUXES REFLECTED ACROSS A LINE OF SYMMETRY INTO
C ANOTHER ANGLE.
DO 49 I=1,40
F3(I)=PBY(1,I)
49 F2(I)=PBX(I,1)
C PBX(I,1) FOR L=9 IS PBX(I,1) FOR L=8
C PBY(1,J) FOR L=9 IS PBY(1,J) FOR L=12
L=10
DO 51 JD=1,40
J=41-JD
DO 51 ID=1,40
I=41-ID
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
PBY(I,J)=SX(I,J,K)-A*PBY(I+1,J)
PBY(I,J)=(PBY(I,J)-C*PBX(I,J+1))/B
P5(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
51 PBX(I,J)=2.*P5(I,J)-PBX(I,J+1)
CALL CKZ(P4,P5)
C PBX(I,1) FOR L=10 IS PBX(I,1) FOR L=7
DO 55 I=1,40
55 F1(I)=PBX(I,1)
L=11

```

```

DO 219 JD=1,40
J=41-JD
DO 219 I=1,40
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
PBY(I+1,J)=(SX(I,J,K)-C*PBX(I,J+1)-B*PBY(I,J))/A
P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
219 PBX(I,J)=2.*P5(I,J)-PBX(I,J+1)
CALL CKZ(P4,P5)
C PBX(I,1) FOR L=11 IS PBX(I,1) FOR L=6
DO 122 I=1,40
122 F4(I)=PBX(I,1)
L=12
DO 57 J=1,40
57 PBY(1,J)=F3(J)
DO 59 JD=1,40
J=41-JD
DO 59 I=1,40
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
PBY(I+1,J)=(SX(I,J,K)-C*PBX(I,J+1)-B*PBY(I,J))/A
P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
59 PBX(I,J)=2.*P5(I,J)-PBX(I,J+1)
CALL CKZ(P4,P5)
C PBX(I,1) FOR L=12 IS PBX(I,1) FOR L=5
DO 64 I=1,40
64 F3(I)=PBX(I,1)
L=3
DO 73 I=1,40
PBX(I,41)=0.
73 PBY(41,I)=0.
DO 10 JD=1,40
J=41-JD
DO 10 ID=1,40
I=41-ID
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
PBY(I,J)=(SX(I,J,K)-A*PBY(I+1,J)-C*PBX(I,J+1))/B
P5(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
10 PBX(I,J)=2.*P5(I,J)-PBX(I,J+1)
CALL CKZ(P4,P5)
DO 13 I=1,40
13 F5(I)=PBX(I,1)
L=4
DO 19 JD=1,40
J=41-JD
DO 19 I=1,40
CALL TOTAL(I,J,K,ST,TOT)
CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)

```

```

PBY(I+1,J)=(SX(I,J,K)-C*PBX(I,J+1)-B*PBY(I,J))/A
P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
19  PBX(I,J)=2.*P5(I,J)-PBX(I,J+1)
    CALL CKZ(P4,P5)
    DO 22 I=1,40
22  F6(I)=PBX(I,1)
    L=8
    DO 67 I=1,40
    PBY(41,I)=0.
67  PBX(I,1)=F2(I)
    DO 126 J=1,40
    DO 126 ID=1,40
    I=41-ID
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I,J)=(SX(I,J,K)-C*PBX(I,J)-A*PBY(I+1,J))/B
    P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
126  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
    CALL CKZ(P4,P5)
C    PBY(1,I) FOR L=8 IS PBY(1,I) FOR L=5
    DO 69 I=1,40
69  F2(I)=PBY(1,I)
    L=7
    DO 70 I=1,40
70  PBX(I,1)=F1(I)
    DO 225 J=1,40
    DO 225 ID=1,40
    I=41-ID
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I,J)=(SX(I,J,K)-C*PBX(I,J)-A*PBY(I+1,J))/B
    P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
225  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
    CALL CKZ(P4,P5)
    L=6
C    PBY(1,J) FOR L=7 IS PBY(1,J) FOR L=6
    DO 71 J=1,40
71  PBX(J,1)=F4(J)
    DO 127 J=1,40
    DO 127 I=1,40
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I+1,J)=SX(I,J,K)-C*PBX(I,J)
    PBY(I+1,J)=(PBY(I+1,J)-B*PBY(I,J))/A
    P5(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
127  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
    CALL CKZ(P4,P5)
    L=5
    DO 72 I=1,40

```

```

    PBY(I,I)=F2(I)
72  PBX(I,1)=F3(I)
    DO 227 J=1,40
    DO 227 I=1,40
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I+1,J)=SX(I,J,K)-C*PBX(I,J)
    PBY(I+1,J)=(PBY(I+1,J)-B*PBY(I,J))/A
    P5(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
227  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
    CALL CKZ(P4,P5)
    L=2
    DO 23 I=1,40
    PBY(41,I)=0.
23  PBX(I,1)=F5(I)
    DO 25 J=1,40
    DO 25 ID=1,40
    I=41-ID
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I,J)=(SX(I,J,K)-C*PBX(I,J)-A*PBY(I+1,J))/B
    P5(I,J)=(PBY(I+1,J)+PBY(I,J))/2.
25  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
    CALL CKZ(P4,P5)
    L=1
    DO 66 I=1,40
66  PBX(I,1)=F6(I)
    DO 27 J=1,40
    DO 27 I=1,40
    CALL TOTAL(I,J,K,ST,TOT)
    CALL CON2(A,B,C,L,DX(I),DY(J),DB2,TOT)
    PBY(I+1,J)=(SX(I,J,K)-C*PBX(I,J)-B*PBY(I,J))/A
    P5(I,J)=(PBY(I,J)+PBY(I+1,J))/2.
27  PBX(I,J+1)=2.*P5(I,J)-PBX(I,J)
C   CALCULATE TOTAL FLUX AT EVERY POINT
    DO 1006 J=1,40
    DO 1006 I=1,40
    IF(P5(I,J))1007,1006,1006
1007 P5(I,J)=0.
1006 P4(I,J)=(P4(I,J)+P5(I,J))/12.
    IF(K-3)1235,1236,1236
1236 DO 1237 J=1,19.
    DO 1237 I=12,13
1237 P4(I,J)=.0
C   IMPROVE SCATTERING SOURCE FOR NEXT GROUP
1235 DO 32 J=1,40
    DO 32 I=1,40
C   STORE THE APPROPRIATE CROSS-SECTIONS IN SCC.
    CALL SCAT(S,SCC,I,J)

```

```

DO 33 L=K,3
N=K+L-(1/K)
33  SX(I,J,L)=SX(I,J,L)+SCC(N)*(P4(I,J)-P(I,J,K))
32  P(I,J,K)=P4(I,J)
C  END OF LOOP
C  WRITE THE TOTAL FLUX ON TAPES 7 AND 8 ALTERNATELY.
C  THIS ALLOWS RESTARTING THE PROGRAM WITH THE BEST
C  AVAILABLE FLUX VALUES. TWO TAPES ARE USED IN CASE
C  THE PROBLEM TERMINATES BEFORE THE LAST WRITE
C  COMMAND CAN BE EXECUTED.
IF(NUM)1162,1162,1163

1162 NQQ=8
NUM=1
GO TO 1164

1163 NQQ=7
NUM=0

1164 II=40
IJ=40
IK=3
WRITE(NQQ)II,IJ,IK,(((P(I,J,K),I=1,II),J=1,IJ),K=1,IK)
END FILE NQQ
REWIND NQQ
WRITE(6,109)NQQ
KTR=1

C  CALCULATE IMPROVED FISSION SOURCE
GO TO 5

C  HAVE INNER ITERATIONS CONVERGED
6  R=(SCE/S1)-1.
IF(ABS(R)-AAA)34,34,357
C  WRITE THE FLUX EVERY 25 ITERATIONS WHETHER OR NOT
C  CONVERGENCE HAS BEEN ACHIEVED.
357 IF(NTP-25)35,358,358
358 WRITE(6,121)R
WRITE(6,100)SCE
WRITE(6,107)NN
WRITE(6,98)
WRITE(6,102)
WRITE(6,103)((I,J,(P(I,J,K),K=1,3),I=1,40),J=1,40)
NTP=0
GO TO 35

34  R=SCE-1.
C  HAVE OUTER ITERATIONS CONVERGED
IF(ABS(R)-.01)36,36,37
36  WRITE(6,104)
NT=0
A=P(1,1,3)
DO 970 K=1,3
DO 970 J=1,40
DO 970 I=1,40

```

```

970 P(I,J,K)=P(I,J,K)/A
      KTR=-1
37  IF(NT)60,377,378
378 IF(NT-50)61,377,377
377 WRITE(6,101)DB2
      WRITE(6,100)SCE
      WRIIE(6,107)NN
      WRITE(6,98)
      WRITE(6,102)
      WRITE(6,103)((I,J,(P(I,J,K),K=1,3),I=1,40),J=1,40)
      NT=0
      IF(DB2)81,65,81
81  IF(KTR)80,65,61
61  IF(NSC)5000,5000,5001
5000 IF(R)5,377,5001
C   ADJUST THE CONTROL ROD BY VARYING THE VOLUME
C   FRACTIONS OF FUEL AND CADMIUM.
5001 CALL ROD(FS,S,ST,VU,VC,NSC,R,QC,QF,QF1,QF2,QC2)
      WRITE(6,108)VU
      IF(VU-.05)60,5011,5011
5011 KTR=0
      NT=NT+1
      GO TO 5
C   AFTER CONVERGENCE, CALCULATE THE FLUX WITH AN
C   INFINITE HEIGHT CORE.
80  DB2=0.
      GO TO 35
65  WRITE(6,113)
98  FORMAT(1H )
99  FORMAT(I8)
100 FORMAT(1H ,21HTOTAL FISSION SOURCE=,E9.3)
101 FORMAT(1H1,14HBUCKLING TERM=,E10.3)
102 FORMAT(39H X Y GROUP 1 GROUP 2 GROUP 3)
103 FORMAT(1H ,2I3,1X,3E11.3)
104 FORMAT(1H1,28HCONVERGED FLUX VALUES FOLLOW)
105 FORMAT(6E12.6)
106 FORMAT(3E12.6)
107 FORMAT(1H ,21HNUMBER OF ITERATIONS=,I8)
108 FORMAT(1H ,24HVOLUME FRACTION OF FUEL=,E10.3)
109 FORMAT(17H FLUX IS ON TAPE ,I3)
113 FORMAT(1H ,18'NORMAL TERMINATION)
119 FORMAT(22A6)
120 FORMAT(12A6)
121 FORMAT(1H ,12H(SCE/S1)-1.=,F15.5)
60  STOP
      END
      SUBROUTINE CON2(A,E,C,L,DX,DY,DB2,TOT)
C   SEPARATE LOOP TO CALCULATE CONSTANT COEFFICIENTS
      XL=L

```

```

        IF(L=4)14,14,1414
14    A=2.*XL-1.
      A=A*3.14159/4.
C     USE THIS VALUE FOR L=1,4
      ETA=.8703883**2
      ETA=1.-ETA
      ETA=SQRT(ETA)
      GO TO 1415
1414  A=3.14159/4.
      A=A*XL
      A=A-9.*3.14159/8.
C     USE THIS VALUE FOR L=5,12
      ETA=.3481553**2
      ETA=1.-ETA
      ETA=SQRT(ETA)
1415  AC=COS(A)
      AS=SIN(A)
      A1=ETA*AC/DX
      A2=ETA*AS/DY
      A3=(DB2+TOT)/2.
      IF(L=4)1416,1416,1417
1416  IF(L=3)30,29,29
1417  IF(L=9)30,29,29
29    A=A1-A2+A3
      B=-A1-A2+A3
      C=2.*A2
      RETURN
30    A=A1+A2+A3
      B=-A1+A2+A3
      C=-2.*A2
      RETURN
      END
      SUBROUTINE FIS(I,J,FS,FCS)
      DIMENSION FS(2,3),FCS(3)
      IF(I.GE.3)GO TO 3000
      IF(J.GE.13)GO TO 3001
      IF(J.GE.5.AND.J.LT.9)GO TO 3001
3000  DO 3002 KK=1,3
3002  FCS(KK)=FS(2,KK)
      RETURN
3001  DO 3003 KK=1,3
3003  FCS(KK)=FS(1,KK)
      RETURN
      END
      SUBROUTINE CKZ(P4,P5)
      DIMENSION P4(40,40),P5(40,40)
      DO 1002 I=1,40
      DO 1002 J=1,40
      IF(P5(I,J))1003,1002,1002

```

```

1003 P5(I,J)=0.
1002 P4(I,J)=P4(I,J)+P5(I,J)
      RETURN
      END
      SUBROUTINE SCAT(S,SCC,I,J)
      DIMENSION S(13,6),SCC(6)
C      THIS SUBROUTINE SEARCHES THROUGH LOGIC STATEMENTS TO LOCATE THE
C      POINT(I,J) IN THE MODEL, AND STORES THE APPROPRIATE SCATTERING
C      CROSS-SECTIONS IN THE MATRIX SCC.
      IF(I.GE.37)GO TO 3015
      IF(I.GE.17.AND.J.GE.33)GO TO 3015
      IF(I.GE.17.AND.I.LE.18.AND.J.GE.29)GO TO 3015
      IF(I.LE.16.AND.J.GE.29)GO TO 3017
      IF(I.LE.18.AND.J.GE.25.AND.J.LE.28)GO TO 3019
      IF(I.GE.17.AND.I.LE.18.AND.J.LE.28)GO TO 3019
      IF(I.GE.21.AND.I.LE.35.AND.J.LE.20)GO TO 3019
      IF(I.GE.23.AND.I.LE.35.AND.J.LE.28)GO TO 3019
      IF(I.EQ.36.AND.J.LE.32)GO TO 3021
      IF(I.GE.19.AND.I.LE.36.AND.J.GE.29)GO TO 3021
      IF(I.GE.19.AND.I.LE.22.AND.J.GE.21)GO TO 3021
      IF(I.GE.19.AND.I.LE.20.AND.J.LE.20)GO TO 3023
      IF(I.GE.15.AND.I.LE.16.AND.J.LE.24)GO TO 3025
      IF(I.GE.11.AND.I.LE.14.AND.J.LE.20)GO TO 3027
      IF(I.GE.11.AND.I.LE.14.AND.J.LE.24)GO TO 3029
      IF(I.GE.9.AND.I.LE.10.AND.J.LE.24)GO TO 3031
      IF(J.GE.21.AND.J.LE.24.AND.I.LE.8)GO TO 3033
      IF(J.GE.17.AND.J.LE.20.AND.I.LE.8)GO TO 3035
      IF(I.GE.3.AND.I.LE.8.AND.J.LE.16)GO TO 3037
      IF(J.LE.4)GO TO 3037
      IF(J.LE.12.AND.J.GE.9)GO TO 3037
      GO TO 3039
C      CONCRETE
      3015 DO 3016 N=1,6
      3016 SCC(N)=S(13,N)
      RETURN
C      WATER
      3017 DO 3018 N=1,6
      3018 SCC(N)=S(12,N)
      RETURN
C      AIR
      3019 DO 3020 N=1,6
      3020 SCC(N)=S(11,N)
      RETURN
C      TEST CELL WALL
      3021 DO 3022 N=1,6
      3022 SCC(N)=S(10,N)
      RETURN
C      AL-H2O 4
      3023 DO 3024 N=1,6

```

```

3024 SCC(N)=S(4,N)
      RETURN
C   AL-H2O 3
3025 DO 3026 N=1,6
3026 SCC(N)=S(3,N)
      RETURN
C   CADMIUM
3027 DO 3028 N=1,6
3028 SCC(N)=S(5,N)
      RETURN
C   ALUMINUM
3029 DO 3030 N=1,6
3030 SCC(N)=S(6,N)
      RETURN
C   AL-H2O 1
3031 DO 3032 N=1,6
3032 SCC(N)=S(1,N)
      RETURN
C   AL-H2O 2
3033 DO 3034 N=1,6
3034 SCC(N)=S(2,N)
      RETURN
C   BE-H2O REFLECTOR
3035 DO 3036 N=1,6
3036 SCC(N)=S(7,N)
      RETURN
C   CORE
3037 DO 3038 N=1,6
3038 SCC(N)=S(8,N)
      RETURN
C   ROD FUEL
3039 DO 3040 N=1,6
3040 SCC(N)=S(9,N)
      RETURN
END
SUBROUTINE TOTAL(I,J,K,ST,TOT)
DIMENSION ST(13,3)
C   THIS SUBROUTINE ALSO SEARCHES THROUGH LOGIC STATEMENTS TO LOCATE
C   THE POINT(I,J), AND STORES THE APPROPRIATE TOTAL CROSS-SECTION
C   FOR ENERGY GROUP (K) IN THE LOCATION TOT
IF(I.GE.37)GO TO 4015
IF(I.GE.17.AND.J.GE.33)GO TO 4015
IF(I.GE.17.AND.I.LE.18.AND.J.GE.29)GO TO 4015
IF(I.LE.16.AND.J.GE.29)GO TO 4017
IF(I.LE.18.AND.J.GE.25.AND.J.LE.28)GO TO 4019
IF(I.GE.17.AND.I.LE.18.AND.J.LE.28)GO TO 4019
IF(I.GE.21.AND.I.LE.35.AND.J.LE.20)GO TO 4019
IF(I.GE.23.AND.I.LE.35.AND.J.LE.28)GO TO 4019
IF(I.EQ.36.AND.J.LE.32)GO TO 4021

```

```

IF(I.GE.19.AND.I.LE.36.AND.J.GE.29)GO TO 4021
IF(I.GE.19.AND.I.LE.22.AND.J.GE.21)GO TO 4021
IF(I.GE.19.AND.I.LE.20.AND.J.LE.20)GO TO 4023
IF(I.GE.15.AND.I.LE.16.AND.J.LE.24)GO TO 4025
IF(I.GE.11.AND.I.LE.14.AND.J.LE.20)GO TO 4027
IF(I.GE.11.AND.I.LE.14.AND.J.LE.24)GO TO 4029
IF(I.GE.9.AND.I.LE.10.AND.J.LE.24)GO TO 4031
IF(J.GE.21.AND.J.LE.24.AND.I.LE.8)GO TO 4033
IF(J.GE.17.AND.J.LE.20.AND.I.LE.8)GO TO 4035
IF(I.GE.3.AND.I.LE.8.AND.J.LE.16)GO TO 4037
IF(J.LE.4)GO TO 4037
IF(J.LE.12.AND.J.GE.9)GO TO 4037
4039 TOT=ST(9,K)
  RETURN
4015 TOT=ST(13,K)
  RETURN
4017 TOT=ST(12,K)
  RETURN
4019 TOT=ST(11,K)
  RETURN
4021 TOT=ST(10,K)
  RETURN
4023 TOT=ST(4,K)
  RETURN
4025 TOT=ST(3,K)
  RETURN
4027 TOT=ST(5,K)
  RETURN
4029 TOT=ST(6,K)
  RETURN
4031 TOT=ST(1,K)
  RETURN
4033 TOT=ST(2,K)
  RETURN
4035 TOT=ST(7,K)
  RETURN
4037 TOT=ST(8,K)
  RETURN
  END
SUBROUTINE ROD(FS,S,ST,VU,VC,NSC,R,QC,QF,QF1,QF2,QC2)
DIMENSION FS(2,3),S(13,6),ST(13,3),QC(6),QF(6),QF1(3),QF2(3),
1QC2(3)
  NSC=NSC+1
  IF(R)1000,1001,1001
1000 Q=1.
  GO TO 1002
1001 Q=-1.
1002 VU=(.5**NSC)*Q+VU
  VC=1.-VU

```

```
DO 1003 N=1,6
C      QF(N) CONTAINS THE ORIGINAL SCATTERING CROSS-SECTIONS
C      FOR THE ROD FUEL, AND QC(N) THOSE FOR CADMIUM.
1003 S(9,N)=QF(N)*VU+QC(N)*VC
      DO 1004 K=1,3
C      FS(1,K) IS ROD FUEL, FS(2,K) IS FIXED FUEL.
C      THE INITIAL FISSION CROSS-SECTIONS ARE STORED IN QF1.
      FS(1,K)=VU*QF1(K)
C      THE INITIAL TOTAL CROSS-SECTIONS FOR ROD FUEL AND
C      CADMIUM ARE STORED IN QF2 AND QC2 RESPECTIVELY.
1004 ST(9,K)=QF2(K)*VU+QC2(K)*VC
      RETURN
      END
$IBMAP INOUT7 4
      ENTRY  •UN07•
•UN07• PZE   UNIT07
UNIT07 FILE  ,A(1),BIN,INOUT,BLK=256,NOLIST,READY
      END
$IBMAP INOUT8 4
      ENTRY  •UN08•
•UN08• PZE   UNIT08
UNIT08 FILE  ,C(1),BIN,INOUT,BLK=256,NOLIST,READY
      END
```

## C SAMPLE INPUT DATA FOR S4C40

2.425	2.425	2.57	2.57	2.57	2.57
2.57	2.57	.2285	.2285	.051	.051
.051	.051	.96	.96	6.985	6.985
.6985	.6985	5.65	5.65	22.5	22.5
22.5	22.5	22.5	22.5	22.5	22.5
22.5	22.5	22.5	22.5	22.5	12.7
22.5	22.5	22.5	22.5		
.23127E+01	.23127E+01	.38545E+01	.38545E+01	.38545E+01	.38545E+01
.38545E+01	.38545E+01	.38545E+01	.38545E+01	.8255E-00	.8255E-00
.69855E+01	.69855E+01	.635E+01	.635E+01	.225E+02	.225E+02
.225E+02	.225E+02				
.15557E-03	.27244E-02	.446E-01			
.904E-04	.127E-02	.304E-01			
.261E+01	.243E+01	.246E+01			
.1E+01	.37E-05	.0E-00			
.001					
.222E-00	.245E-02	.156E-03	.119E-00	.422E-02	.1765E-00
.2759E-00	.186E-01	.135E-05	.373E-00	.320E-01	.878E-00
.249E-00	.108E-01	.775E-06	.250E-00	.186E-01	.539E-00
.227E-00	.524E-02	.363E-06	.151E-00	.372E-02	.255E-00
.277E-00	.2747E-03	0.E-00	.226E-00	.462E-03	.322E-00
.2127E-00	.3493E-03	0.E-00	.856E-01	.5989E-03	.84E-01
.446E-00	.122E-01	.266E-06	.720E-00	.194E-01	.930E-00
.378E+00	.484E-01	.3571E-05	.8439E+00	.8331E-01	.217E+01
.38652E-00	.50856E-01	.3751E-05	.888E-00	.88886E-01	.2297E+01
.274E-00	.294E-02	.173E-06	.571E-00	.243E-02	.667E-00
.92E-04	.589E-09	.0E-00	.249E-03	.0E-00	.381E-03
.483E-00	.791E-01	.585E-05	.132E+01	.136E-00	.351E+01
.243E-00	.197E-01	.138E-05	.442E-00	.3E-01	.828E-00
.22452E-00	.12426E-00	.1891E-00			
.29478E-00	.40551E-00	.892E-00			
.26036E-00	.26945E-00	.542E-00			
.232457E-00	.15555E-00	.268E-00			
.2868E-00	.4285E-00	.1355E+03			
.2134E-00	.8687E-01	.962E-01			
.46067E-00	.73928E-00	.931E-00			
.427E-00	.93337E-00	.871E-01			
.43783E-00	.98225E-00	.23746E+01			
.277E-00	.57637E-00	.787E-00			
.9525E-04	.2492E-03	.438E-03			
.5627E-00	.14627E+01	.353E+01			
.2634E-00	.4734E-00	.848E-00			

8

ALL INPUT DATA HAS BEEN READ CORRECTLY

Glossary of Computer Program Symbols

A	: Azimuthal angle ( $\phi_2$ )
AAA	: Convergence criterion ( $\epsilon$ )
DB2	: Vertical leakage term ( $DB_Z^2$ )
DX	: Increment in the x-direction
DY	: Increment in the y-direction
FS	: Fission cross-section
NN	: Number of iterations
P	: Total neutron flux ( $\bar{N}(\vec{r})$ )
PBX	: Angular flux ( $N_{\vec{r}, j}$ )
PBY	: Angular flux ( $N_{i, \vec{r}}$ )
S	: Scattering cross-section
SCE	: Total fission source ( $\lambda$ )
ST	: Total cross-section
SUM	: Fission source in one cell $\left[ \sum_{g=1}^3 \nu_g \sum_f^g N_g(\vec{r}) \right]$
SX	: Scattering source in one cell
V	: Average number of neutrons per fission ( $\nu$ )
VC	: Volume fraction of cadmium in rod fuel
VU	: Volume fraction of fuel in rod fuel
X	: Group source ( $\chi$ )

BUCKLING TERM= 0.478E-02  
 TOTAL FISSION SOURCE=0.170E 02  
 NUMBER OF ITERATIONS= 5

X	Y	GROUP 1	GROUP 2	GROUP 3
1	1	0.688E-02	0.373E-03	0.653E 00
2	1	0.740E-02	0.398E-03	0.720E 00
3	1	0.772E-02	0.418E-03	0.740E 00
4	1	0.781E-02	0.425E-03	0.738E 00
5	1	0.767E-02	0.416E-03	0.737E 00
6	1	0.690E-02	0.374E-03	0.665E 00
7	1	0.572E-02	0.311E-03	0.546E 00
8	1	0.361E-02	0.186E-03	0.399E-00
9	1	0.198E-02	0.897E-04	0.298E-00
10	1	0.175E-02	0.857E-04	0.275E-00
11	1	0.164E-02	0.824E-04	0.359E-01
12	1	0.159E-02	0.799E-04	0.
13	1	0.155E-02	0.776E-04	0.
14	1	0.151E-02	0.755E-04	0.135E-03
15	1	0.119E-02	0.639E-04	0.538E-01
16	1	0.717E-03	0.440E-04	0.121E-01
17	1	0.509E-03	0.323E-04	0.446E-02
18	1	0.424E-03	0.262E-04	0.436E-02
19	1	0.348E-03	0.211E-04	0.394E-02
20	1	0.278E-03	0.179E-04	0.321E-02
21	1	0.231E-03	0.150E-04	0.239E-02
22	1	0.203E-03	0.128E-04	0.192E-02
23	1	0.134E-03	0.830E-05	0.120E-02
24	1	0.561E-04	0.343E-05	0.413E-03
25	1	0.195E-05	0.370E-06	0.107E-04
26	1	0.237E-06	0.407E-07	0.822E-05
27	1	0.870E-05	0.442E-06	0.865E-04
28	1	0.460E-05	0.324E-06	0.110E-04
29	1	0.823E-06	0.363E-07	0.916E-05
30	1	0.364E-05	0.222E-06	0.397E-04
31	1	0.192E-05	0.148E-06	0.786E-05
32	1	0.401E-06	0.616E-07	0.656E-05
33	1	0.387E-05	0.195E-06	0.255E-04
34	1	0.307E-06	0.427E-07	0.782E-05
35	1	0.314E-06	0.874E-08	0.159E-04
36	1	0.310E-05	0.981E-07	0.194E-04
37	1	0.140E-06	0.123E-07	0.418E-05
38	1	0.776E-06	0.538E-07	0.951E-05
39	1	0.122E-06	0.110E-07	0.384E-05
40	1	0.279E-06	0.266E-07	0.864E-05
1	2	0.554E-02	0.354E-03	0.622E 00
2	2	0.709E-02	0.382E-03	0.685E 00
3	2	0.755E-02	0.409E-03	0.722E 00
4	2	0.780E-02	0.423E-03	0.747E 00
5	2	0.765E-02	0.416E-03	0.737E 00

8 40	0.123E-08	0.416E-09	0.197E-06
9 40	0.300E-08	0.125E-09	0.431E-04
10 40	0.645E-07	0.311E-03	0.865E-05
11 40	0.137E-06	0.533E-09	0.177E-05
12 40	0.171E-06	0.619E-08	0.391E-04
13 40	0.194E-06	0.696E-08	0.588E-04
14 40	0.209E-06	0.730E-08	0.386E-04
15 40	0.895E-07	0.368E-08	0.105E-04
16 40	0.139E-07	0.858E-09	0.103E-04
17 40	0.391E-08	0.283E-09	0.158E-05
18 40	0.103E-07	0.734E-09	0.144E-05
19 40	0.105E-07	0.224E-08	0.511E-05
20 40	0.389E-07	0.157E-08	0.301E-04
21 40	0.151E-07	0.129E-08	0.366E-05
22 40	0.212E-07	0.779E-09	0.263E-05
23 40	0.548E-08	0.243E-09	0.843E-06
24 40	0.396E-08	0.215E-09	0.326E-06
25 40	0.599E-08	0.345E-09	0.851E-06
26 40	0.244E-08	0.641E-09	0.382E-06
27 40	0.538E-08	0.349E-09	0.889E-06
28 40	0.132E-09	0.221E-09	0.382E-06
29 40	0.101E-07	0.390E-09	0.888E-06
30 40	0.213E-09	0.268E-09	0.377E-06
31 40	0.164E-07	0.504E-09	0.865E-06
32 40	0.253E-09	0.930E-09	0.555E-06
33 40	0.192E-07	0.651E-09	0.990E-06
34 40	0.220E-09	0.406E-09	0.422E-06
35 40	0.165E-07	0.536E-09	0.138E-05
36 40	0.291E-09	0.287E-09	0.770E-06
37 40	0.115E-07	0.526E-09	0.125E-05
38 40	0.142E-09	0.184E-09	0.453E-06
39 40	0.115E-07	0.662E-09	0.121E-05
40 40	0.133E-09	0.219E-09	0.463E-06

VOLUME FRACTION OF FUEL= 0.500E-00

FLUX IS ON TAPE 8

FLUX IS ON TAPE 7

FLUX IS ON TAPE 8

FLUX IS ON TAPE 7

FLUX IS ON TAPE 8

VOLUME FRACTION OF FUEL= 0.250E-00

FLUX IS ON TAPE 7

FLUX IS ON TAPE 8

VOLUME FRACTION OF FUEL= 0.125E-00

## Appendix B

The GAM-I Code

The GAM-I code (Refs 10,11) is a computer program written for the IBM 7094 to generate multigroup cross-sections using the  $P_1$  or  $B_1$  approximation. It will calculate these constants for as many as 32 fast groups. Available as output are diffusion coefficients, a full scattering matrix which includes inelastic and  $(n-2n)$  scatter, age, and constants for one fast group.

There are three options for calculating cross-sections; one may use the  $P_1$  equations, use the  $B_1$  equations, or read in a flux spectrum. In the  $P_1$  approximation (Ref 14:336) the scattering frequency function and flux are expanded in Legendre polynomials, and the  $P_1$  equations are then integrated over space to yield two energy dependent equations. These two equations are then solved iteratively for the flux and current, and the fine (sixty-eight) group cross-sections are weighted appropriately.

The  $B_1$  approximation (Ref 9) uses a Fourier transform of the one-dimensional Boltzmann equation, with the transformed flux and frequency function expanded in Legendre polynomials. Given the wave number "k", the transformed equations for flux and current are solved iteratively. This method is the most accurate for homogeneous materials. It assumes that the energy spectrum of the source term is independent of position. To get some

idea of the dependence of the  $B_1$  approximation on the wave number used,  $k$  was taken as the square-root of the transverse buckling  $(B_y^2)^{\frac{1}{2}}$  and then of the total buckling  $(B_t^2)^{\frac{1}{2}}$ . The change in cross-sections for water was very slight.

The 16-group cross-sections given by Yiftah et al (Ref 17) were used for comparison with the GAM-I values; both use the Cranberg fission spectrum for U-235 (Ref 7). Since GAM-I will accept any fast energy groups (up to 32), Yiftah's group structure was used by the program to generate a set of cross-sections. Since Yiftah uses a different flux spectrum, the results were not expected to be identical. The two sets were in fairly good agreement, however, especially at very high energies; hence, more faith could be put in the GAM-I values, and this code was used exclusively.

Appendix C

Personal Correspondence

The two letters in this appendix were received from workers at Los Alamos Scientific Laboratory, New Mexico. They are self-explanatory in their content, the first referring to Ref 13 and the second to Ref 2, as applied to the derivation of S<sub>4</sub>C<sub>4</sub>O; both are valuable as sources of additional detail for the above references..

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 LOS ALAMOS, NEW MEXICO 87544

IN REPLY  
 REFER TO: T-4

January 4, 1965

Lt. Lee H. Livingston  
 AFTT Box 3368  
 Wright-Patterson AFB  
 Ohio 45433

Dear Lt. Livingston:

In reply to your letter of December 2, 1964 I would like to offer the following comments:

Question 1: Concerning the objects  $\hat{f}$  and  $\hat{\beta}$  of Chapter 4. On page 75 (LA-2595),  $\hat{f}$  and  $\hat{\beta}$  are defined by Eqn. 4.8, 4.9 and the text immediately below. The crucial point here is the recognition that the transport equations can be transformed into an "exact differential" form, which, when integrated over the independent variable, depends only upon the end points (Condition A). How is this done? If one uses Eqn. (4.7) as a guide in writing the transport equation terms (see Table 4.1), then the application of Condition A (Eqn. 4.8) becomes a trivial identity when the integration is performed over the applicable independent variable. Eqn (4.9) is likewise an identity. However, its difference approximation is only to first order.

We could write, equally well, Eqn 4.10, as

$$\begin{aligned}
 T(x)N(x) &\equiv \frac{\partial}{\partial x} f(x)N(x) = f(x) \frac{\partial N}{\partial x} + N(x) \frac{\partial f}{\partial x} \\
 &\approx f_{ave} \frac{N_{l+1} - N_l}{\Delta x} + \frac{N_{l+1} + N_l}{2} \left( \frac{\partial f}{\partial x} \right)_{ave} \\
 &= \hat{f}(N_{l+1} - N_l) + \frac{N_{l+1} + N_l}{2} \hat{\beta}
 \end{aligned}$$

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where  $f(x)$  is some analytic function occurring in the transport equation and  $\hat{f} = \frac{f_{ave}}{\Delta x}$ ,  $\hat{\beta} = \left( \frac{\partial f}{\partial x} \right)_{ave}$ . However, we have still to determine an appropriate form for  $\hat{f}$  and  $\hat{\beta}$  consistent with the rest of the approximation.

The philosophy invoked here is that either way of writing

$$T(x)N(x) = \frac{\partial}{\partial x} f(x)N(x) = f(x) \frac{\partial N}{\partial x} + N(x) \frac{\partial f}{\partial x}$$

in difference approximation must be equivalent in its application to the remitting equations.

In Eqn. (4.11), the integral is approximated by the averaged value of the integrand (using the right hand side of Eqn. (4.9)) with an unknown (but determinable) weight function  $w$ . Eqn. (4.12) performs the same order of approximation but with the alternate form (Condition A). Thus, one can obtain the recursion relationship given by Eqn. (4.13).

This technique is just that required to guarantee exact conservation of neutrons, a crucial requirement for a transport code.

Question 2: Equivalence of Eq. (4.32) of LA-2595 and Eqn. (7) of Carlson's Numerical Solutions of Neutron Transport Problems. They are exactly the same equations except for a notational change in the angular coordinates, labeling of the interval subscripts and use of the diamond differencing assumption Eqn. (4.18). We now indicate the details for clarification: Write Eqn. (4.32) (ignoring 4th and 5th terms for the moment) as

$$\frac{\mu}{V_i} (A_{i+1} N_{i+1} - A_i N_i) + \frac{\eta}{V_i} (B_{j+1} N_{j+1} - B_j N_j) + \frac{\xi}{V_i} (C_{k+1} N_{k+1} - C_k N_k) + \sigma_i N'_i = S_i \quad (1)$$

Eqn. (7) is given by

$$\begin{aligned} & (\eta \cos \bar{\omega}_m + \eta A_i \sin \bar{\omega}_l / \Delta_j + h_i + \bar{\mu} \Delta_i / \Delta_k) N_i \\ & + (-\eta \cos \bar{\omega}_m + \eta A_i \sin \bar{\omega}_l / \Delta_j + h_i + \bar{\mu} \Delta_i / \Delta_k) N_{i-1} \\ & - 2(\eta A_i \sin \bar{\omega}_l (\Delta_j) N_{j-1} - 2\bar{\mu} \frac{\Delta_i}{\Delta_k} N_{k-1}) = \Delta_i S_i \end{aligned} \quad (2)$$

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Eqn. (1) and (2) first of all differ in the angular variable notation, so let me identify

$$\begin{aligned}
 \mu' &= \eta \cos \bar{\phi}_m \\
 \eta' &= \eta \sin \bar{\phi}_s \\
 \xi' &= \bar{\mu} \\
 \eta &= \sqrt{1 - \mu^2}
 \end{aligned} \tag{3}$$

Then Eqn (2) becomes

$$\begin{aligned}
 &(\mu' + \eta' \frac{\Delta_i}{\Delta_j} + \xi' \frac{\Delta_i}{\Delta_k} + h_i) N_i + (-\mu' + \eta' \frac{\Delta_i}{\Delta_j} + \xi' \frac{\Delta_i}{\Delta_k} + h_{i-1}) N_{i-1} \\
 &- 2\eta' \frac{\Delta_i}{\Delta_j} N_{j-1} - 2\xi' \frac{\Delta_i}{\Delta_k} N_{k-1} = \Delta_i S_i
 \end{aligned} \tag{4}$$

Next divide out  $\Delta_i$ , and introduce  $V_i \equiv V_{ijk} \equiv \Delta_i \Delta_j \Delta_k$ , yielding

$$\begin{aligned}
 &\left( \frac{\mu'}{V_i} \Delta_j \Delta_k + \frac{\eta'}{V_i} \Delta_i \Delta_k + \frac{\xi'}{V_i} \Delta_i \Delta_j + \frac{h_i}{\Delta_i} \right) N_i + \left( -\frac{\mu'}{V_i} \Delta_j \Delta_k + \frac{\eta'}{V_i} \Delta_i \Delta_k + \frac{\xi'}{V_i} \Delta_i \Delta_j + \frac{h_i}{\Delta_i} \right) N_{i-1} \\
 &- \frac{2}{V_i} \eta' \Delta_i \Delta_k N_{j-1} - 2 \frac{\xi'}{V_i} \Delta_i \Delta_j N_{k-1} = S_i
 \end{aligned} \tag{5}$$

Now regroup the terms to obtain

$$\begin{aligned}
 &\frac{\mu'}{V_i} \Delta_j \Delta_k (N_i - N_{i-1}) + \frac{\eta'}{V_i} \Delta_i \Delta_k (N_i + N_{i-1} - 2N_{j-1}) \\
 &+ \frac{\xi'}{V_i} \Delta_i \Delta_j (N_i + N_{i-1} - 2N_{k-1}) + c_i \frac{(N_i + N_{i-1})}{2} = S_i
 \end{aligned} \tag{6}$$

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If we utilize the diamond difference condition Eqn. (4.25)

$$2N'_i = N_i + N_{i+1} = N_j + N_{j+1} = N_\ell + N_{\ell+1} = N_m + N_{m+1} \dots \quad (7)$$

in the above equation, then

$$N_i + N_{i-1} = N_j + N_{j-1} \Rightarrow N_i + N_{i-1} - 2N_{j-1} = N_j - N_{j-1} \quad (8)$$

$$N_i + N_{i-1} = N_k + N_{k-1} \Rightarrow N_i + N_{i-1} - 2N_{k-1} = N_k - N_{k-1}$$

$$\frac{N_i + N_{i-1}}{2} = N'_i$$

and (6) becomes

$$\frac{\mu'}{V_i} \Delta_j \Delta_k (N_i - N_{i-1}) + \frac{\eta'}{V_i} \Delta_i \Delta_k (N_j - N_{j-1}) + \frac{\xi'}{V_i} \Delta_i \Delta_j (N_k - N_{k-1}) + \sigma_i N'_i = S_i \quad (9)$$

Now identify the area factors  $A_i = A_{i-1} = \Delta_j \Delta_k$ ,  $B_j = B_{j-1} = \Delta_i \Delta_k$ ,  $C_k = C_{k-1} = \Delta_i \Delta_j$  and (9) becomes

$$\frac{\mu'}{V_i} (A_i N_i - A_{i-1} N_{i-1}) + \frac{\eta'}{V_i} (B_j N_j - B_{j-1} N_{j-1}) + \frac{\xi'}{V_i} (C_k N_k - C_{k-1} N_{k-1}) + \sigma_i N'_i = S_i \quad (10)$$

which is just Eqn. (1) offset by one interval. Notice that Carlson uses  $\Delta_i = r_i - r_{i-1}$  whereas LA-2595 uses  $\Delta_i = r_{i+1} - r_i$ .

Question 3: Why do the fourth and fifth terms of Eqn. (4.32) drop out?  
 By definition for x, y, z geometry, Eqn. (4.33),

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$$A_i = \Delta y \Delta z \neq \text{function of } x$$

$$B_j = \Delta x \Delta z \neq \text{function of } y$$

$$C_k = \Delta x \Delta y \neq \text{function of } z$$

The subscript convention announced was  $i \in x$ ,  $j \in y$ ,  $k \in z$ . Thus,  $A_i = A_{i+1}$  for all  $i$ ,  $B_j = B_{j+1}$  for all  $j$ ,  $C_k = C_{k+1}$  for all  $k$ . The cross section perpendicular to a given coordinate is independent of that coordinate since the local geometry is flat. This is true only for  $(x, y, z)$ , since no curvature terms are present in the operators.

Question 4: Concerning values of  $W_{lm}$  to be used. The condition on  $W_{lm}$  is  $\sum w_{lm} = 1$  when summed over the lowest order symmetry of the sphere used (octant, hemisphere etc.). In Table 4.5 this is two octants, and hence  $\frac{1}{2} W_{lm}$  is used. If a hemisphere is necessary  $\frac{1}{4} W_{lm}$  is used. If the full sphere  $(x, y, z)$  is used,  $\frac{1}{8} W_{lm}$  is the appropriate quantity. The Appendix A may be of some help. Be careful of direction cosines when taking the full sphere!

Finally some general remarks. Your proposed project is very ambitious. You should realize that about a factor 25 increase in running time is involved in going from  $rz$  to  $xyz$  geometry due to increased number of directions and additional required space points.

We include an  $rz$  Fortran Code listing for your information and the associated flow diagrams. Tapes and cards can be made available if that would help.

If we may be of further assistance, please let us know.

Sincerely yours,

*Clarence E. Lee*  
Clarence E. Lee  
T-Division

CEL/ml

Enc: Fortran code listing  
Flow diagrams

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IN REPLY  
REFER TO:

T-1

March 1, 1965

Mr. Lee H. Livingston  
5466 Access Road  
Dayton, Ohio - 45431

Dear Mr. Livingston:

Bengt Carlson has asked me to answer your questions of 18 February. From your letter I gather that you are referring to the paper in Volume XI of Proceedings of Symposia in Applied Mathematics. The selection of the discrete angles  $\varphi$  is described in paragraph 5 on page 224 of the proceedings. In equation 6 on page 228 the angles  $\varphi_m$  and  $\varphi_l$  are both the same angles, i.e.,  $\varphi_{ml}$ . The absence of the subscript is a convention. The orientation of these angles is sketched for  $n=4$  on the enclosed page.

The material described in the above reference is badly out of date. For a description of methods in use in 1964 I suggest you study Los Alamos Scientific Laboratory Report LA-2996. Angular orientations and direction sets are described in LA-3186, a copy of which is enclosed.

If you have more questions, please feel free to write to me.

Sincerely yours,

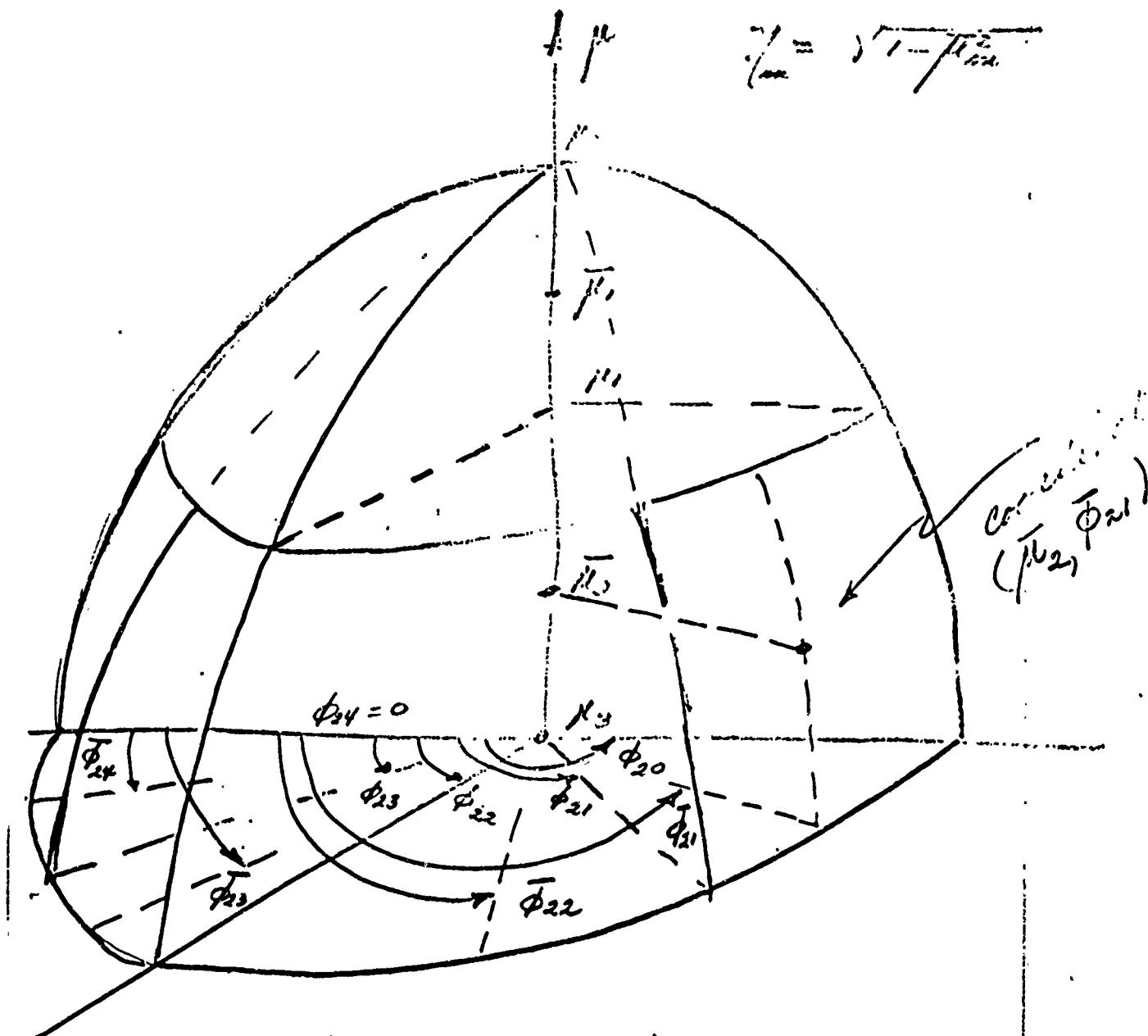


Dr. K. D. Lathrop

KDL:sv  
encl: 1 graph + LA-3186

$$1' = \beta'$$

$$\gamma = \beta' - \beta_{\text{ext}}$$



The  $m$  subscript refers to level along the  $\mu$  axis, while the  $l$  subscript measures angle of rotation about the  $\mu$  axis. The point on the unit sphere with coordinates  $\bar{\mu}_2, \bar{\phi}_3$ , is shown. From the tables for  $n=4$   $\bar{\mu}_2 = 0.348$   
 $\bar{\phi}_{21} = 7\pi/8$  68

VITA

Lee Haltom Livingston was born on 5 August 1939 in Memphis, Tennessee, the son of Thomas Elliott Livingston and Katherine Stewart Livingston. Upon graduation in 1957 from Central High School in Memphis, he enlisted in the United States Navy and served for two years until July 1959 when he received a fleet appointment to the United States Naval Academy. In June 1963 he graduated from the Academy with the degree of Bachelor of Science and received his commission as Lieutenant in the USAF. He came to the Air Force Institute of Technology in July of that year.

Permanent address: 4780 Dee Road

Memphis Tennessee 38117